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ANISOTROPIC TENSILE PROBABILISTIC FAILURE CRITERION FOR COMPOSITES

by

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Anisotropic Tensile Probabilistic Failure Criterion for Composites

by

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ABSTRACT

A probabilistic failure criterion is needed to quantitatively predict reliability in critical applications,such as man-safe, deep-sea and air structures, and as an objective function for use in optimum design. Composites are multi-phased and anisotropic, which gives rise to failure in different modes with different probabilistic occurrences that are dependent on the applied stress tensor. Statistical representation of combined stress failures is practically impossible. Probabilistic modeling must be based on the failure modes. This investigation examines the underlying features required in a probabilistic failure criterion for unidirectional fiber-composite structures via Monte Carlo simulations. The interdependencies of the intrinsic strengths (associated with uniaxial loadings) and of the failure modes in a composite structure under combined tensile loading are elucidated. The joint distribution function for composite failure due to a proportional loading regime is derived starting from the representation of the physical failure process in Boolean operations which, in turn, is represented by probability functions. Specific forms of the probability functions for different failure modes are suggested.

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TABLE OF SYMBOLS

$C_{ij}(\sigma_1,\sigma_2)$	Coupling function which defines the dependency of failure mode M_i on σ_j.
$f_{X_i}(\sigma_i)$	Uniaxial failure pdf of the stress component σ_i.
$f_{X_1,X_2}(\sigma_1',\sigma_2')$	Joint failure pdf in the transformed stress space
$f_{(\mathcal{F}_1)_1,(\mathcal{F}_2)_2}(\sigma_1,\sigma_2)$	Joint failure pdf in the physical stress space.
$f_{(\mathcal{F}_i)_i\mid(\mathcal{F}_j)_j}(\sigma_1\mid\sigma_2)$	Conditional probability of failure due to failure mode M_i given occurrence of failure mode M_j.
$F_c(\sigma_1,\sigma_2)$	Joint CDF for the composite in biaxial tensile combined stress.
$F_{X_i}(\sigma_i)$	Uniaxial CDF of the stress component σ_i.
$F^*(\sigma)$	Linearized form of the CDF, $F(\sigma)$.
$\underset{\sim}{F}_i$	Intrinsic strength vector associated with failure mode M_i.
$\underset{\sim}{F}_i(\sigma_1,\sigma_2;X_i,\Theta_i)$	Deterministic failure criterion for a given specimen which represents failure mode M_i in the stress space.
$(\mathcal{F}_i)_j$	Component of the intrinsic strength vector, $\underset{\sim}{F}_i$, corresponding to the σ_j axis in the stress space.
m_{12}	Slope of the stress vector in the biaxial stress space.
M_i	Failure mode associated with stress component, σ_i.
$R_c(\sigma_1,\sigma_2)$	Reliability of the composite in biaxial tensile combined stress.
$\underset{\sim}{S}$	Representation of the applied stress tensor in vector form.
X_i	Uniaxial strength associated with failure mode M_i.
α	Weibull shape parameter.
β	Weibull scale parameter.
δ	Ineffective length of fiber.component of a composite.

σ_b Dry bundle (no matrix) tensile strength of constituent fiber in a composite.

σ_i Stress vector component manifest in failure mode M_i.

Θ_i Set of coupling parameters for the deterministic failure criterion, $\mathfrak{F}_i(\sigma_1,\sigma_2;X_i,\Theta_i)$.

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I. INTRODUCTION

Many structural composite applications require a quantitative determination of reliability which, in turn, requires a probabilistic failure criterion. Justification for a quantitative reliability analysis may take many forms. Man-safe applications, such as aircraft and deep submergence vessel components, and inaccessible applications, such as satellite components, require design and certification of the composite to a specified reliability level prior to service. Large complex composite structures, for which proof testing would be economically impractical, require extensive preliminary reliability analysis to ensure a zero-reject rate once in service. Reliability is frequently used as an objective function in the optimization of design and repair of composite structures. It may also be used in the maintenance of composite structures in order to identify critical inspection requirements. For any case, quantitative reliability analysis of a composite structure requires a mathematical function which specifies the stress or strain levels below which the probable risk of failure is acceptable. This function is a probabilistic failure criterion.

Failure occurs in composite structures manifest in different modes which result from the material heterogeneity and anisotropy inherent in composite materials. The unidirectional fiber composites used in high performance structural applications consist of many strong stiff fibers embedded in a relatively ductile matrix. The fibers account for virtually all of the structural strength along their longitudinal axis, but they make a negligible contribution to the structural strength in the orthogonal in-plane (transverse) direction. Thus, the structural strength of the composite in the direction coinciding with the longitudinal axis of

the fibers is typically significantly less than the structural strength in the transverse direction. In addition, failure due to the component of the internal stress tensor in the longitudinal direction of the fibers alone is a sequential process exacerbated by the localized clustering of fiber breaks. Failure due to the component of the internal stress tensor in the transverse direction alone is characterized by crack propagation within the matrix binder. These two failure modes were observed to be different physical processes.

The mathematical function may be formulated to represent the failure states or it may be formulated to model the physical process. The former is termed the phenomenological failure criterion and the latter, the mechanistic failure criterion. A phenomenological failure criterion only represents the failure state of a composite; it does not explicitly attempt to model the underlying failure mechanisms themselves [Ref. 1]. A phenomenological failure criterion may be viewed as a mathematical transfer function which relates the excitation function (in this case, the applied stress tensor) to the material response (failure). As extended to the failure characterization of anisotropic composites, the phenomenological failure criterion is intended to assist in experimental design i.e., to facilitate the interpolation, correlation and retrieval of experimental observations. In general, it does not address the case where strength variation is described by a statistical distribution.

Since data is the basis of statistics, a probabilistic characterization of composite failure requires an extensive experimental data base if it is to be considered phenomenologically based. Any failure criterion may be represented geometrically as a limiting envelope in the stress space and the shape of the failure envelope for a phenomenological failure criterion cannot be completely known

until experiments are performed for all possible states of combined stress. In order to represent statistical variation of strength as in the case of a probabilistic failure characterization, the failure envelope is replaced by a set of failure contours where each contour represents the locus of all stress states having a specific probability of failure. For the probabilistic failure model to be solely based on phenomenological observations, closely spaced series of experiments must be performed and failure probabilities determined along every possible loading path in the stress space. Such an extensive data base does not exist for composite materials and the acquisition of such a data base for a composite is economically infeasible.

Rational formulation of an anisotropic probabilistic failure criterion for composites, therefore, requires an understanding of the essence of the possible failure mechanisms. Then formulation of the probabilistic failure criterion may be reduced to expressing the mathematical combination of the probabilistic models for each of the possible failure mechanisms using Boolean operations. If the mathematical nature of this combination of models can be analytically determined, then the shape of the aforementioned failure contours may be completely visualized once the statistical parameters of the probabilistic models governing each individual failure mechanism are experimentally determined. This combined analytical model significantly reduces the number of experimental measurements and simplifies the data base required to fully describe the failure contours in the stress space

A deterministic failure criterion is an expedient representation of the averaged micromechanical failure processes. An anisotropic deterministic failure criterion presumes that the material is macroscopically homogeneous and anisotropic;

therefore, its intrinsic strength in any given direction is uniform. This implies that when the component of the internal stress tensor in a given direction is nonuniform, then failure will occur at the location along that direction where the component of the internal stress tensor is greatest. In actuality, the intrinsic strength of the material in any given direction is not uniform due to material heterogeneity and the statistical distribution of fiber intrinsic strengths. If the internal stress component in a given direction is nonuniform, then the most probable location of failure is where the stress-strength ratio is greatest, which is not necessarily the location of highest stress. A deterministic failure criterion cannot address this phenomenon. In addition, the assumption of uniform strength in any given direction would preclude the observed sequential failure process in composites under longitudinal loading because all fibers would break simultaneously if their intrinsic strengths were uniform. When the inherent expediency in a deterministic failure criterion is not appropriate for the specific application, as in high reliability or large structures, suitable extension is required.

One such extension is an anisotropic probabilistic failure criterion. A probabilistic failure criterion acknowledges the existence of variable intrinsic strength within a material by treating the intrinsic strength as a random variable whose fractional probability of realization is describable by a particular statistical distribution function. With this model, the strength of the material is presumed to be nonuniform and unknown until realized in failure due to an applied stress. If the distribution function and its parameters are known, then the probabilistic failure criterion can provide the reliability of the structure as the applied stress is increased up to a given value. The scope of this investigation will be limited to the tensile domain.

The characterization of probabilistic failure in composites requires knowledge of the applied stress tensor and, if the stress is nonuniform, knowledge of the spatial location of failure within the structure. Both of these requirements may be motivated and illustrated through the use of statistical simulation.

The objective of this investigation is twofold. First, this investigation will motivate the necessary characteristics of the probabilistic failure criterion through the use of numerical simulation for the cases of uniform and nonuniform stress states. This will be presented in Chapter III. Second, this investigation will identify the mathematical formalism needed in the formulation of an anisotropic probabilistic failure criterion of general applicability and develop explicit expressions of the criterion for several cases of mechanistic dependence between failure modes. This will be presented in Chapter IV.

II. APPROACH

Tensile failure in idealized composite structures will be numerically simulated and statistical data for each structure will be compiled to aid in the recognition of the fundamental parameters of probabilistic composite failure. Then, a formalized formulation of a probabilistic failure criterion of general application will be presented.

It is impractical to perform actual experiments to collect the requisite data for probabilistic characterization of tensile composite failure. This is due primarily to the necessity of accounting for all of the many possible permutations involved in the sequential failure of composites. Furthermore, experimental techniques are not yet available to accurately identify internal fiber failure sites and the critical location of fiber failure clusters from which catastrophic failure of a structure initiates. Numerical simulation, as opposed to experimentation, provides an expedient means for compiling the statistical data necessary to visualize and identify the underlying parameters of probabilistic composite failure.

A formalized formulation of a probabilistic failure criterion is necessary; the mathematical requirements and constraints must be clearly identified so that major limitations and restrictions which apply to the formulation are known. There are many potential constraints which must be considered in the formulation and application of a general probabilistic failure criterion. These include, but are not necessarily limited to: interdependency of uniaxial intrinsic strength components of a composite material, dependency of failure probability on the external loading regime and mechanistic interdependence of failure modes. A purely ad hoc

formulation without regard for mathematical formalism cannot possibly identify the level of applicability or the limitations of the result.

III. PROBLEM IDENTIFICATION VIA SIMULATION

Numerical simulation illustrates many of the physical and statistical phenomena which characterize probabilistic composite failure. The structural strength variability as manifest in testing by measurement of the external load is not necessarily the same as the internal strength variability which is intrinsic to the composite; the difference between them is dependent on the internal stress distribution within the structure. When the internal stress distribution is nonuniform (heterogeneous), the structure does not necessarily fail at the location with the lowest intrinsic strength or the highest stress but rather the location where the stress-to-strength ratio is the highest. When the structure is analyzed as a spatially two-dimensional array of structural elements, the width size effect, which is governed by the element grid used in the analysis, and the effect of elemental load sharing must also be considered in order to obtain realistic simulated results. Finally, in geometrically complex structures, uniaxial loading may result in decidedly multiaxial internal stress distributions for which a one-dimensional characterization of probabilistic failure is wholly inadequate. This becomes the motivation for the formulation of a probabilistic failure criterion for multiaxial combined stress distributions.

A. BACKGROUND

Current models of univariate (uniaxial) probabilistic composite failure did not explicitly address the effect of heterogeneous stress distributions on the characterization of failure in the model structures. In this investigation, the one-dimensional characterization of probabilistic composite failure is numerically

simulated using model geometries similar to those used in actual materials testing and a uniaxial failure criterion. Since the intrinsic strength distribution within physical test specimens is governed by the random strength distribution of the material, the strength distributions of the model structures are generated using Monte Carlo simulation. The model geometries and elemental arrays within the model structures are varied to illustrate the effect of heterogeneous stress distributions, elemental grid definition and internal load sharing on structural failure characterization. A uniaxial probabilistic failure model is shown to be inadequate for representing the failure behavior of a composite under a combined stress distribution and, hence, a combined stress probabilistic failure criterion is necessary.

Rosen investigated uniaxial probabilistic failure in a composite for a uniform (homogeneous) stress distribution using a model consisting of a one-dimensional array of structural elements with the structure represented as a chain of elements.[Ref. 2] He observed that structural failure for this model was a "weakest link" or series process in which the failure of the weakest element constituted failure of the structure and that the elemental intrinsic strengths were determined by the statistical distribution of imperfections among the elements. This led to the concept of the length size effect on structural strength; a structure divided into a greater number of smaller elements will tend to have fewer imperfections in a given element and, hence, the elemental strengths will appear to increase as the number of elements is increased. Since the statistical strength of the entire structure must be independent of the number of elements, the scale parameter of the elemental strength distribution must be corrected for the number of elements used to account for this effect. Rosen also concluded that, over a small, but finite,

length about a fiber break, the broken fiber in ineffective in carrying the uniaxial load and that, over this length, the load is transmitted via the matrix through shear to the adjacent fibers. This length is known as the ineffective length, δ.

Harlow and Phoenix investigated uniaxial probabilistic failure for a homogeneous stress distribution for a composite structure modeled as a two-dimensional elemental configuration by reducing the model to an effective one-dimensional element array with the structure represented as a chain of element bundles.[Ref.3 and 4] They identified the failure process for this model as being a modified "weakest link" or series-parallel process in which the failure of the weakest bundle of elements constituted structural failure. Based on the concept of ineffective length, they developed the local load sharing rule for the chain-of-bundles model by accounting for all permutations of element failure sequences within a bundle. Through numerical analysis of the local load sharing rule for different statistical strength distributions and bundle sizes, they concluded that a chain of element bundles possessed a lower probability of failure at low applied load than a chain of single elements of equal length; this is the effect of local load sharing. In addition, Harlow and Phoenix numerically established the existence of a width size effect for the chain-of-bundles model; as the number of elements per bundle is increased for a given structure, the likelihood of bundles containing one or more weak elements is increased and the bundle strengths appear to decrease. As in the case of the length size effect, the scale parameter of the elemental strength distributions must be corrected--in this case, for the number of elements per bundle--for the structural strength to be independent of the element distribution.

Geometrically complex composite structures, such as plates with holes or inclusions, are typified by clearly heterogeneous stress states, even when the

external loading is uniaxial. The currently available models do not explicitly illustrates the effect of a heterogeneous stress distribution on the one-dimensional characterization of composite failure.

In this simulation, composite test specimens are modeled as elastic and homogeneous in composition, but anisotropic in terms of responses to applied deformation. The fiber-matrix heterogeneity implies strength anisotropy and is incorporated into the structural model by way of the load sharing process. Representative specimen geometries, which are typically used in materials testing, distill the effects of the internal stress distribution and spatial location of failure on the characterization of probabilistic failure.

A specimen structure is visualized as a spatial array of structural elements. These elements, as in the physical case, would have different intrinsic strengths characterized by the statistical distribution of strength for the material. The intrinsic strengths of the individual elements are generated by Monte Carlo simulation. The Monte Carlo technique used in this simulation consists of assigning to each element a fractional ranking of its intrinsic strength given by randomly generated numbers between zero and one, noninclusive. The intrinsic strengths of each element may then be computed from its respective fractional ranking for a set of relevant statistical parameters of the underlying strength model for the parent composite. The specific parametric influence of the strength variability is varied to explore the effect of internal stress distribution on the external strength statistics as measured by the applied load.

A univariate failure criterion in terms of uniaxial stress is employed in the simulation. An element fails when the internal stress on the element is greater than or equal to its intrinsic strength.

A one-dimensional element array similar to that used by Rosen, combined with a heterogeneous stress distribution, is used to elucidate the phenomenological differences between the overall structural strength variability and the intrinsic strength variability as affected by the nature of the internal stress distribution within the structure. A two-dimensional elemental array with a heterogeneous stress distribution is used to show the effect of the width size effect on the overall structural strength variability.

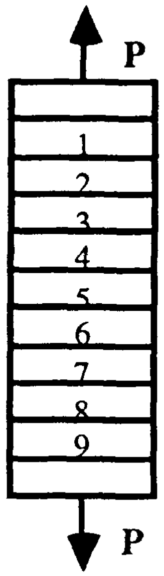
The numerical simulation is straightforward for the one-dimensional elemental distribution because only the failure of single elements constitutes the failure of the entire structure. The simulation becomes more complex for elemental distributions of two or more dimensions because all of the failure sequences must then be considered in the numerical model. Furthermore, for internal stress distributions of two or more dimensions, all of the failure processes must be accounted for. Numerical simulation of composite failure for a two-dimensional elemental distribution due to a one-dimensional internal stress distribution can be accomplished with a uniaxial probabilistic failure criterion and is discussed in the remaining section of this chapter. Simulations of composite failure due to an internal stress distribution of two or more dimensions cannot be adequately accomplished with a uniaxial failure criterion and require the formulation of a combined stress failure criterion, which will be performed in Chapter IV. Once this failure criterion is formulated, characterization and numerical simulation of composite failure for stress distributions of two or more dimensions may be performed through the use of a post-processor for finite element stress analysis results.

B. SIMULATION PROCEDURE

Four structural models were investigated in the simulation to illustrate the influence of heterogeneous stress distributions, internal load sharing and the width size effect on uniaxial composite failure behavior. These models are schematically represented in Figure 3-1. Because the uniaxial failure criterion for the model structures defined the realization of failure in terms of the internal random variable, intrinsic strength, it was essential that a transfer function be identified in order to transform the internal random variable to the external random variable.

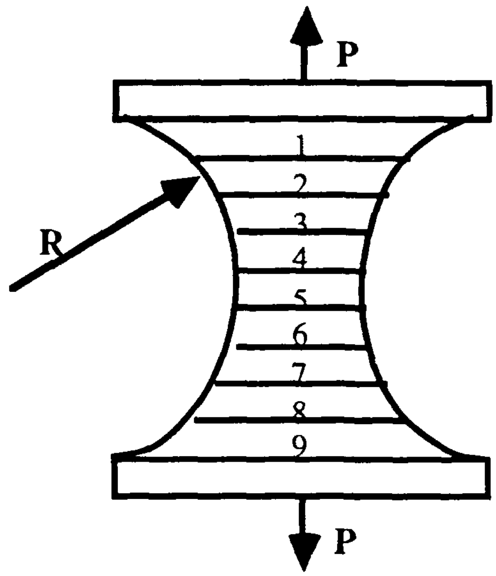
The internal stress distribution was governed by the spatial geometry of the specimen model. All specimen models featured a uniform thickness. For the heterogeneous stress distribution, a model approximating a "dogbone" specimen was used in which the width of the specimen was reduced by two circular arcs with a given radius and colinear centers. A homogeneous stress distribution was created with the "dogbone" specimen geometry by using a radius several orders of magnitude greater than that used for the heterogeneous stress distribution. A model approximating a square plate with a concentric circular hole was used to simulate failure in a complex geometry. The internal stress distribution within the plate for a unit uniaxial external load was determined using the ADINA finite element code. The normal stress components--other stress components were not used in this simulation--along the direction of loading for each element were compiled and incorporated into the simulation through a stress multiplier array, which converted the external applied stress on the plate model into the individual elemental internal stresses.

For the one-dimensional element distribution, the "dogbone" specimen models were initially divided into nine elements of equal length. By using an equal number



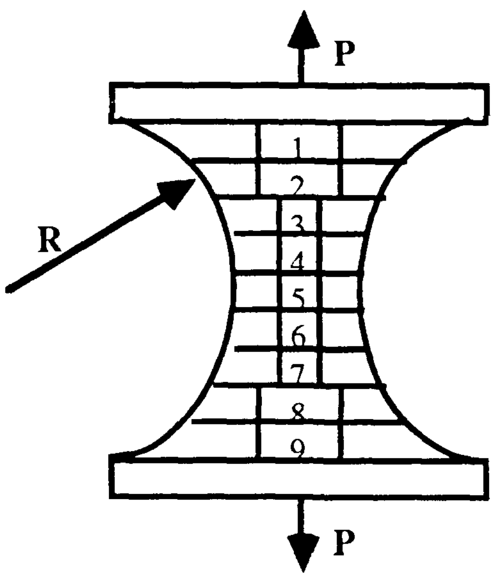
(a)

One-Dimensional Element Distribution
Homogeneous Stress Distribution



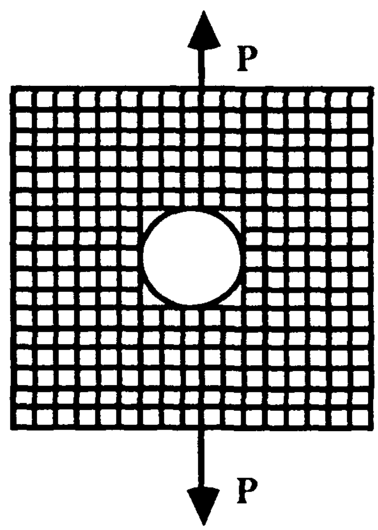
(b)

One-Dimensional Element Distribution
Heterogeneous Stress Distribution



(c)

Two-Dimensional Element Distribution
Heterogeneous Stress Distribution



(d)

Plate with Concentric Circular Hole
24 x 24 Elements

Figure 3-1. Structural models used in the numerical simulation.

of elements of the same length, the length size effect was eliminated; however, the strength for any arbitrary element length could be accounted for through the use of size effect calculations. Thus, the model is analytically valid for any arbitrary element length so long as any stress gradient within a given element is negligible. The one-dimensional "dogbone" specimen model was converted to a two-dimensional model by laterally subdividing each of the original elements into three smaller elements of equal width, the original elements thus becoming bundles of elements. The plate model consisted of 500 square elements arranged in a 24-by-24 element array less the 76 elements which approximated the hole.

Elemental strengths were calculated from randomly generated fractional strength rankings. The Type III, or Weibull, cumulative distribution function (CDF) was used as a transfer function between intrinsic strength and fractional strength ranking

$$F(\sigma)=1-\exp\left[-\left(\frac{\sigma}{\beta}\right)^{\alpha}\right] \tag{3-1}$$

$F(\sigma)$ was the fractional ranking of σ, and α and β were the shape (variability) and scale (magnitude) parameters, respectively, of the CDF. The internal stress, σ, was the realization of the random elemental intrinsic strength; at element failure, σ equaled the intrinsic strength. When the Weibull CDF was inverted so that the failure stress, σ, was given in terms of the fractional strength ranking, F,

$$\sigma(F)=\exp\left\{\left(\frac{1}{\alpha}\right)\ln\left[-\ln(1-F)\right]+\ln(\beta)\right\} \tag{3-2}$$

Equation (3-2) was used in the Monte Carlo simulation for all models to calculate the random elemental intrinsic strengths.

An advantage of numerical simulation in the acquisition of statistical data is that it allows otherwise indiscernible internal physical processes to be visualized in terms of mathematical calculations. For this reason, the simulations for this investigation were performed using the Microsoft Excel™ spreadsheet since each computational step could be visually displayed.

Each simulation essentially followed the same computational sequence. Fractional rankings of elemental strength were randomly generated and assigned to each element of a given specimen. Using the input parameters of the statistical strength distribution, the elemental intrinsic strengths were calculated using Equation (3-2). At this point, a specimen with random strength is simulated. To simulate the loading of the specimen,.an initial value for the external load was imposed on the specimen and the internal stress on each element was calculated based on the cross-sectional area established by the specimen geometry. The value for the internal stress was compared to the respective intrinsic strength for each element; if the stress equaled or exceeded the intrinsic strength for an element, then the element was identified as having failed. For the one-dimensional elemental distribution, this was equivalent to the structure having failed; for the two-dimensional elemental distribution, the load on the adjacent elements in the bundle was increased in accordance with a local load sharing rule where the load originally carried by the failed element is equally distributed to its contiguous neighboring elements. After the load redistribution, each elemental failure criterion is evaluated. Once the failure criterion had been evaluated for all intact elements with no failure indications at a given external load value, the load value

was incremented and the process repeated until structural failure was indicated. If structural failure was indicated, the desired failure parameters were recorded and the simulation repeated with a new set of fractional strength rankings.

The numerical results of the simulation are intended to simulate actual experimental data. For this reason, the simulation process must mirror the essence of the actual physical failure process. A random variable for which data is simulated must be experimentally measurable. Conventional experimental testing procedures use the external failure load as the random variable. Direct experimental measurement of the internal failure stress, i.e., the intrinsic strength, would require the knowledge of the exact internal failure site within the structure; this is difficult to implement experimentally. Therefore, comparison of external failure load to internal failure stress motivates the relevancy of the analysis presented in subsequent sections of this chapter.

C. SIMULATION RESULTS

The effects of homogeneous and heterogeneous internal stress distributions on failure location are presented in the form of histograms for the one-dimensional element distribution and for different intrinsic strength variabilities. Graphical representation of the comparison of internal and external random variables for homogeneous and heterogeneous stress distributions and different intrinsic strength variabilities for the one-dimensional element distribution is also presented. Statistical strength comparisons are presented in the form of linearized Weibull cumulative distribution functions (CDF) of failure load to illustrate the differences in statistical parameters for each data set. Finally, spreadsheet formulas and the controlling macros for the simulations are presented in Appendix A.

The linearized Weibull CDF for the failure load, P, is obtained by isolating the exponential term in Equation (3-1) and taking the logarithm of both sides of the equation:

$$\ln\left[1-F(\sigma)\right] = -\left(\frac{\sigma}{\beta}\right)^{\alpha} \tag{3-3}$$

The linearized CDF, $F^*(\sigma)$, is formed by multiplying both sides of Equation (3-3) by -1 and taking the logarithm once more:

$$F^*(\sigma) = \ln\left\{-\ln\left[1-F(\sigma)\right]\right\} = \alpha \ln\left(\frac{\sigma}{\beta}\right) \tag{3-4}$$

Substituting the failure load, P, as the independent variable into Equation (3-4) gives the linearized CDF for the failure load, P:

$$F^*(P) = \ln\left\{-\ln\left[1-F(P)\right]\right\} = \alpha \ln\left(\frac{P}{\beta}\right) \tag{3-5}$$

The Weibull CDF in linearized form becomes a straight line with slope equal to the shape parameter, α. The value of F^* takes on the value of zero when P equals the shape parameter for the failure load, β, or (P/β) equals one.

1. Significance of Failure Location and Stress Distribution

Failure data was simulated for the specimen models with one-dimensional elements, respectively with homogeneous and heterogeneous stress distributions. The effects to be observed are the external failure load magnitude and variability, and the distribution of spatial locations of failure. Parameters examined are the intrinsic strength variabilities through varying the Weibull shape parameter, α, between high and low values while the Weibull scale parameter, β, for all

for all specimens was held constant.. For each simulated test, failure location, failure load and failure stress were recorded.

When the internal stress distribution is homogeneous, all of the elemental stresses are equal. Therefore, failure of the structure is caused by the failure of the element with the lowest intrinsic strength. Since the intrinsic strengths are randomly distributed among the elements, the location of failure is also random. This random distribution of failure location is independent of the strength variability. These results are illustrated in Figures 3-2(a) and 3-2(b).

The homogeneous stress distribution also implies constant elemental cross-sectional areas. Since stress is defined as load per unit area and the elemental cross-sectional areas are constant, the failure load may be transformed to equivalent data for failure stress by a single scalar value, $(\text{Area})^{-1}$; the location of failure is not required to perform the transformation. Figures 3-3(a) and 3-3(b) show this result for different strength variabilities. This single-valued scalar transformation is independent of the strength variability as shown in Figure 3-4 in which the results of the high and low variability data sets are merged.

For a heterogeneous stress distribution, the most likely failure location is that element with the highest internal stress with low strength variability accentuating this effect. Structural failure in this case does not depend on the lowest elemental strength, but on the lowest elemental stress-to-strength ratio. An element with a high internal stress will generally fail unless it has a significantly high strength and an element with a lower stress has a significantly lower strength. As the strength variability decreases, the elemental intrinsic strengths approach a uniform deterministic value and, in the limiting case, failure will always be caused by the failure of the element with the highest internal stress. Figures 3-5(a) and 3-5(b)

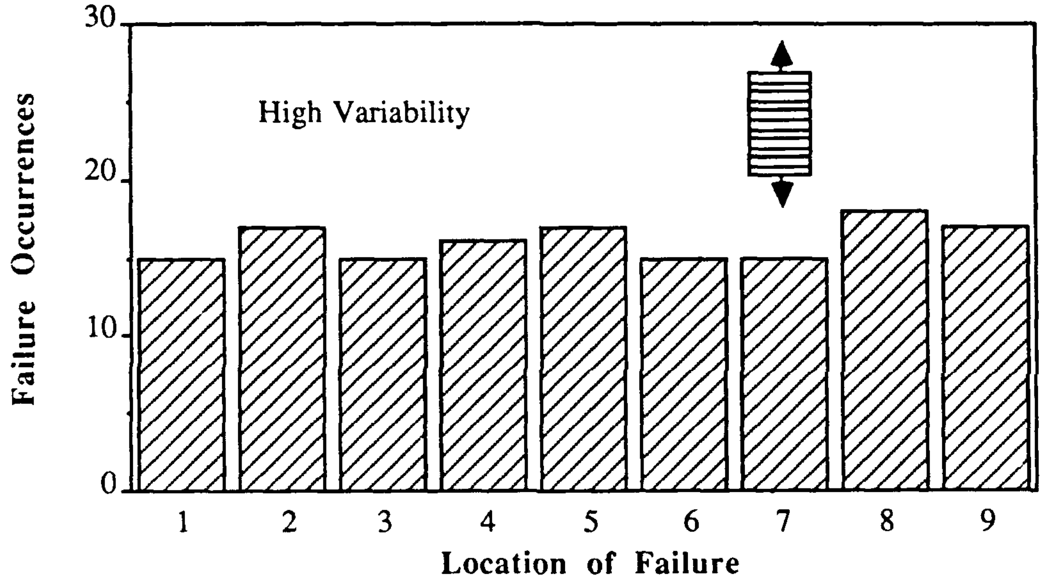


Figure 3-2(a). Distribution of failure location for homogeneous stress and high strength variability.

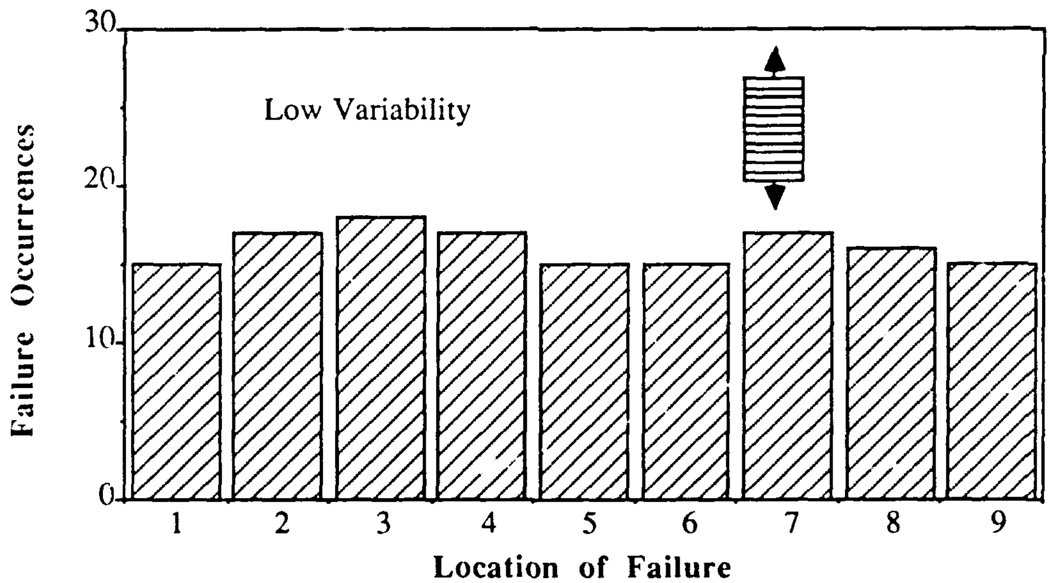


Figure 3-2(b). Distribution of failure location for homogeneous stress and high strength variability.

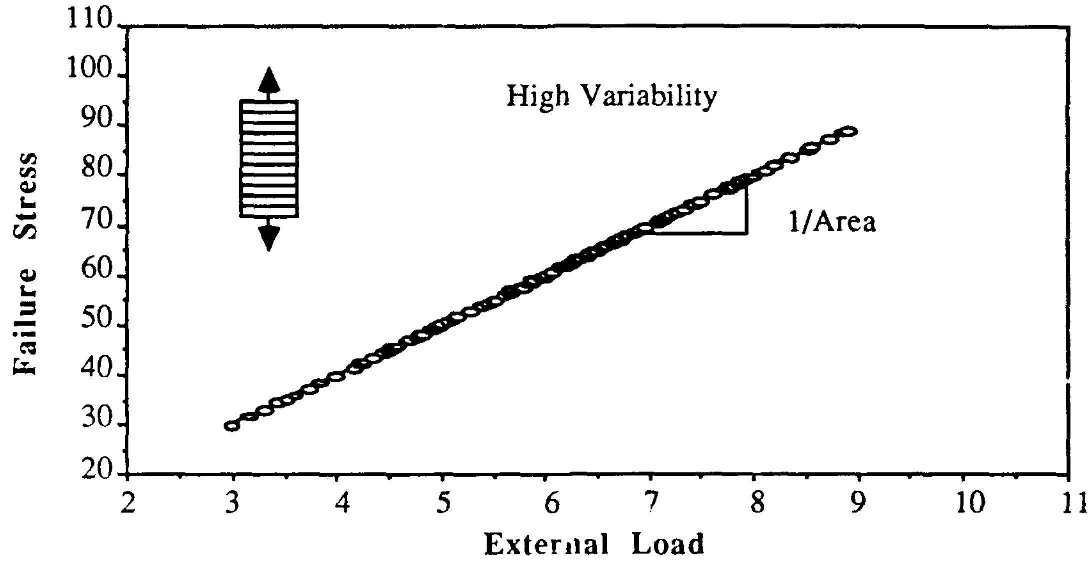


Figure 3-3(a). Failure stress vs. external failure load for homogeneous stress and high strength variability.

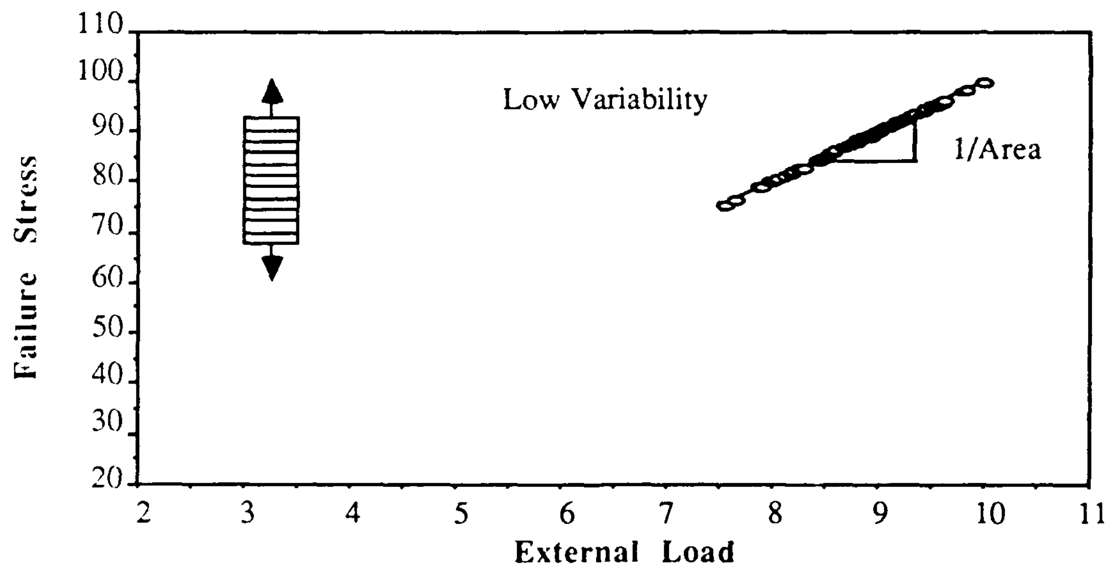


Figure 3-3(b). Failure stress vs. external failure load for homogeneous stress and low strength variablity.

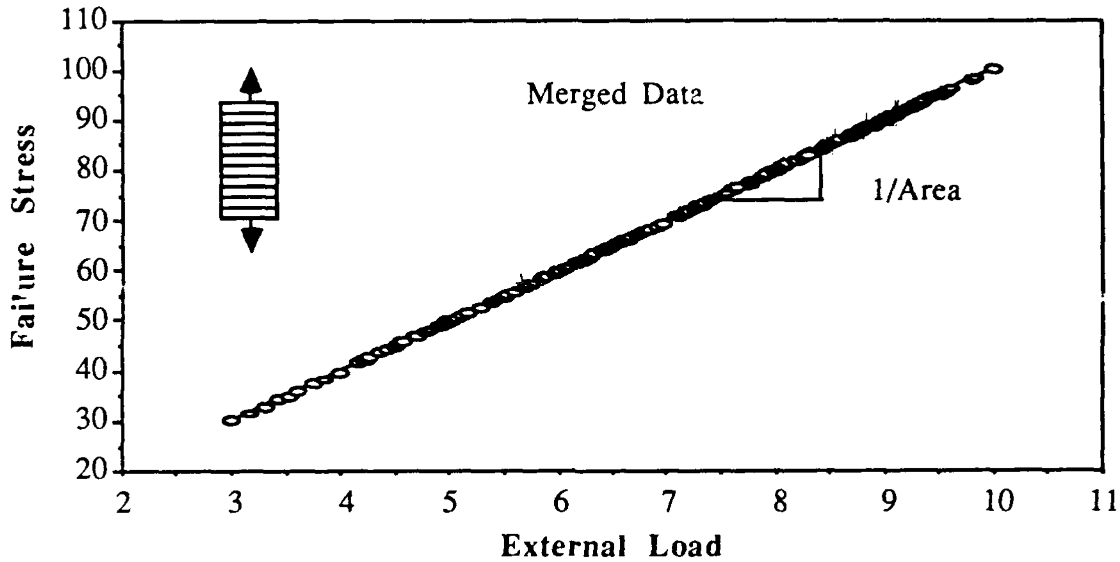


Figure 3-4. Failure stress vs. external failure load for homogeneous stress (merged data sets).

illustrate the distributions of failures by element for high and low strength variabilities.

The heterogeneous stress distribution implies a variation in elemental cross-sectional areas. Therefore, failure load may not be transformed to equivalent data for failure stress by a single scalar value but by a variable scalar value, (Failure Area)$^{-1}$, where the failure area is the cross-sectional area of the failed element. Clearly the failure location must be known in this case for the transformation to be performed. Interpretation of statistical data based on the external failure load without regard for the failure location will result in an erroneous estimation of the statistical parameters of the intrinsic strength distribution for the structure (Figure 3-6). Figures 3-7(a) and 3-7(b) show the variable scalar transformation for high and low strength variability and Figure 3-8

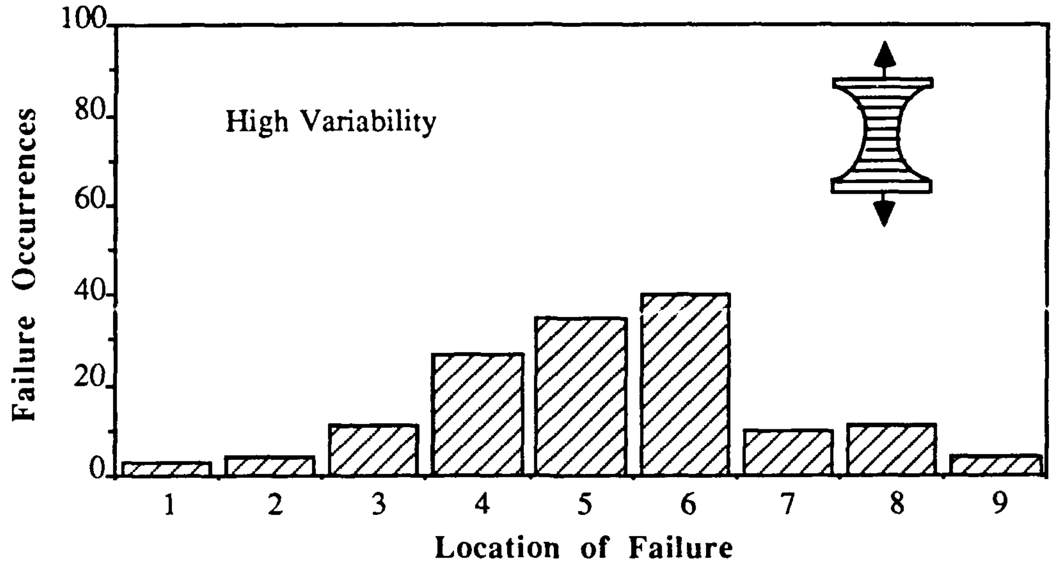


Figure 3-5(a). Distribution of failure location for heterogeneous stress and high strength variability.

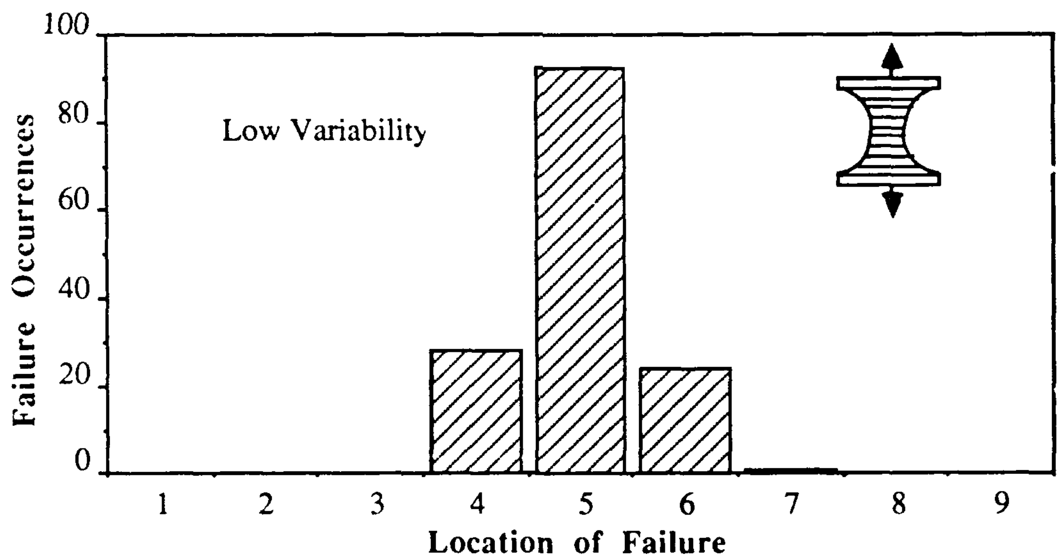


Figure 3-5(b). Distribution of failure location for heterogeneous stress and low strength variability.

shows that the variable scalar transform is independent of the strength variability. When the failure location for the combined data sets is considered, accurate estimation of the statistical parameters of the intrinsic strength distribution for the structure is obtained (Figure 3-9).

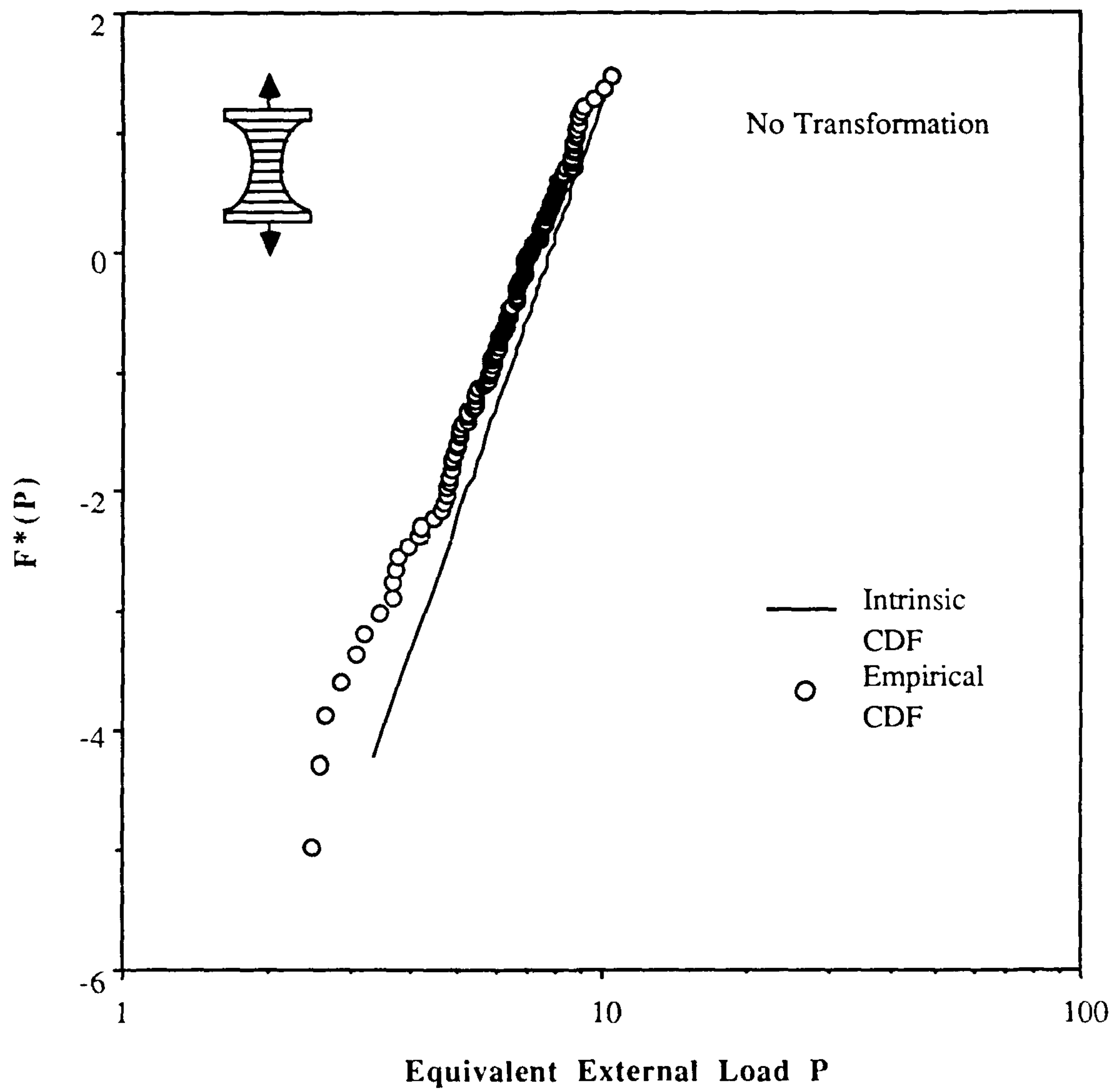


Figure 3-6. Comparison of intrinsic and empirical CDF for heterogeneous stress and high strength variablity without transformation.

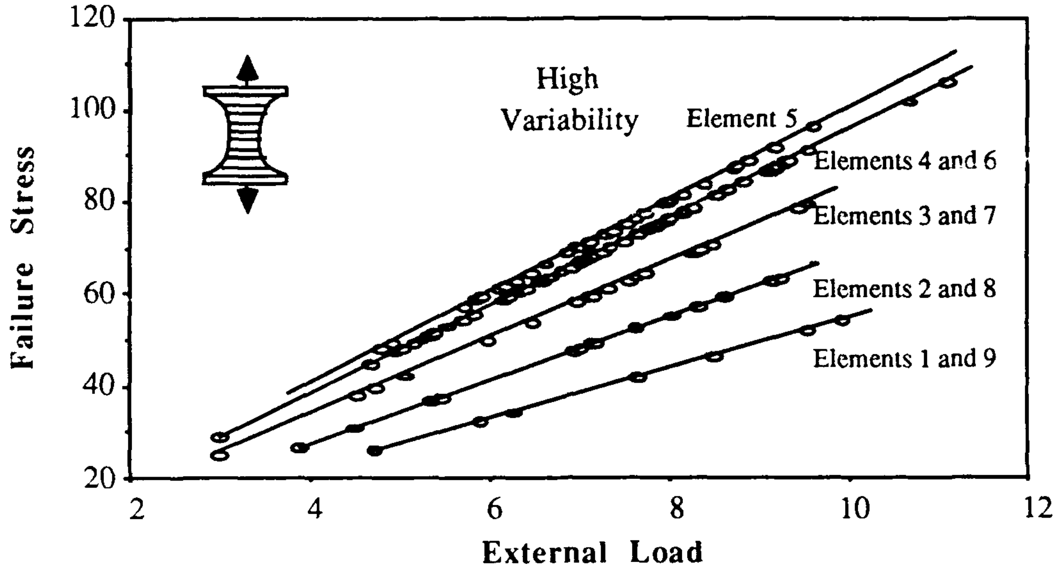


Figure 3-7(a) Failure stress vs. external failure load for heterogeneous stress and high strength variability.

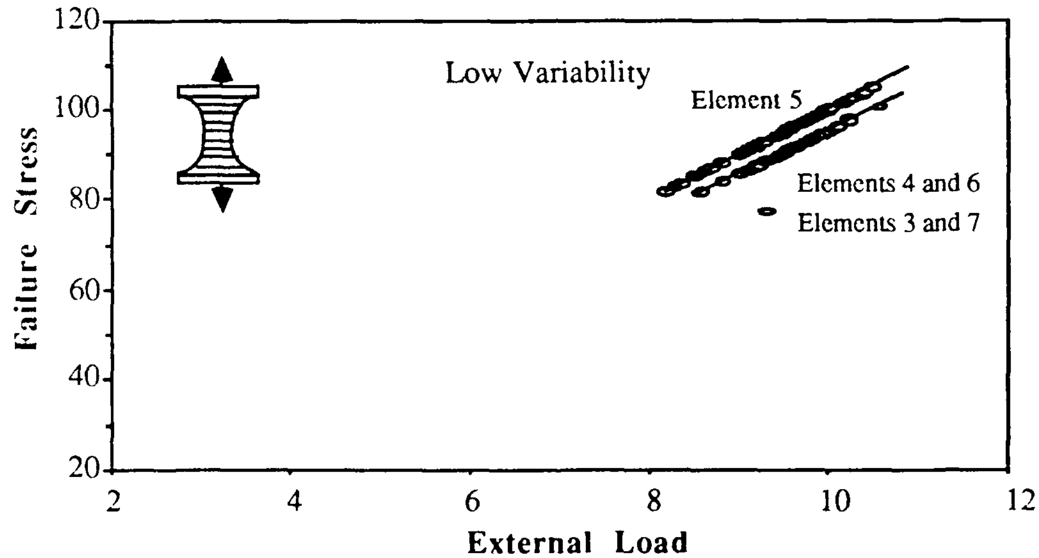


Figure 3-7(b) Failure stress vs. external failure load for heterogeneous stress and high strength variability.

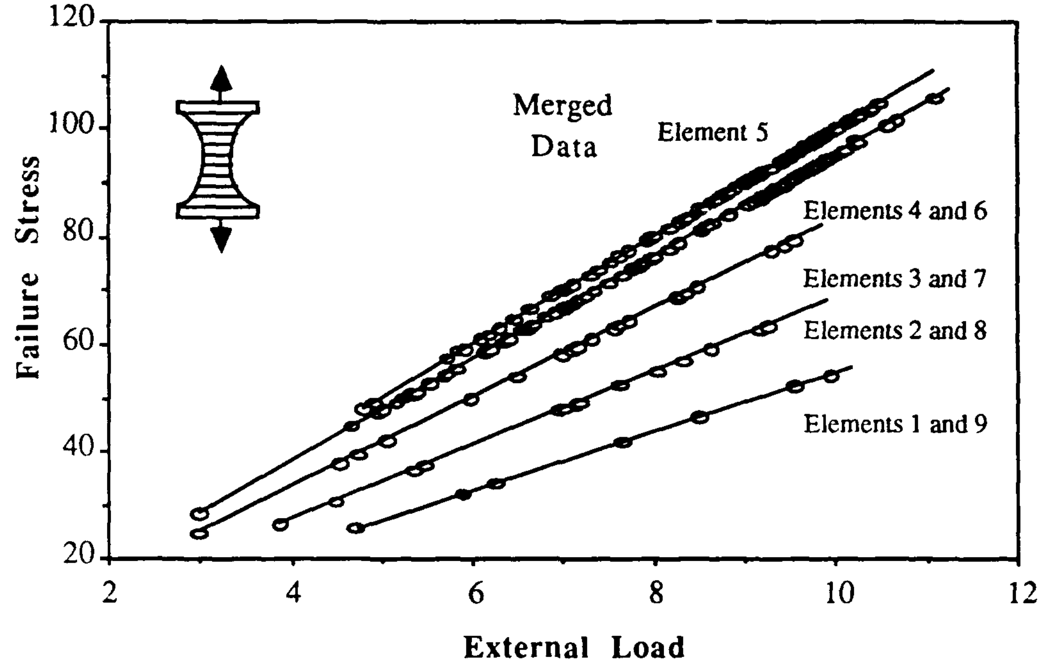


Figure 3-8. Failure stress vs. external failure load for heterogeneous stress (merged data sets).

The empirical strength CDF based on the external failure load converges quickly to the intrinsic CDF for the material in the case of low strength variability; when the strength variability is high, convergence is much slower and more simulations are required. The convergences are illustrated for the homogeneous stress distribution in Figures 3-10(a) and 3-10(b) for low variability, and 3-11(a) and 3-11(b) for high variability.

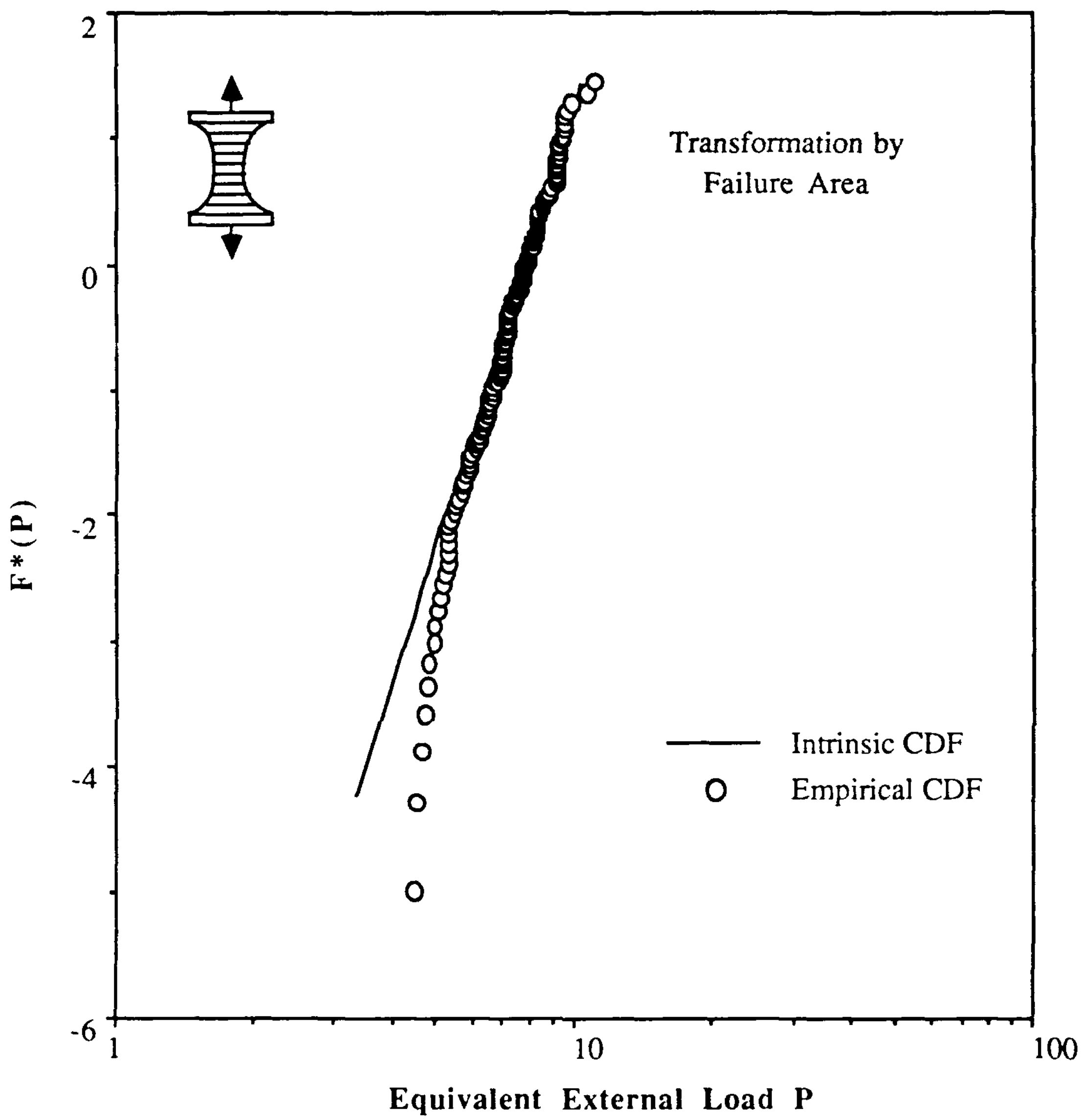


Figure 3-9. Comparison of intrinsic and empirical CDF's for heterogeneous stress and high strength variablity with transformation.

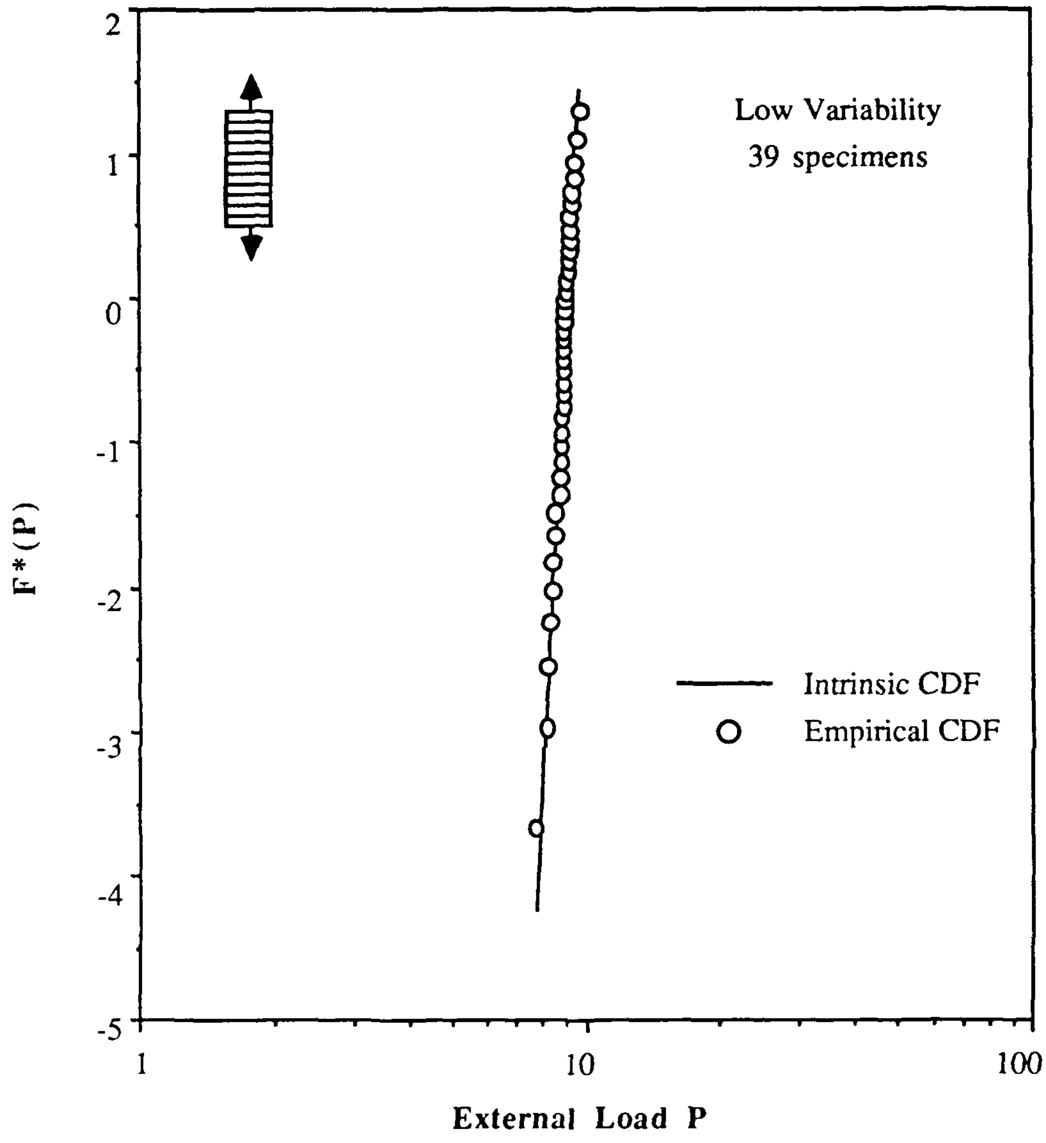


Figure 3-10(a). Comparison of intrinsic and empirical CDF's for homogeneous stress and low strength variability after 39 simulations.

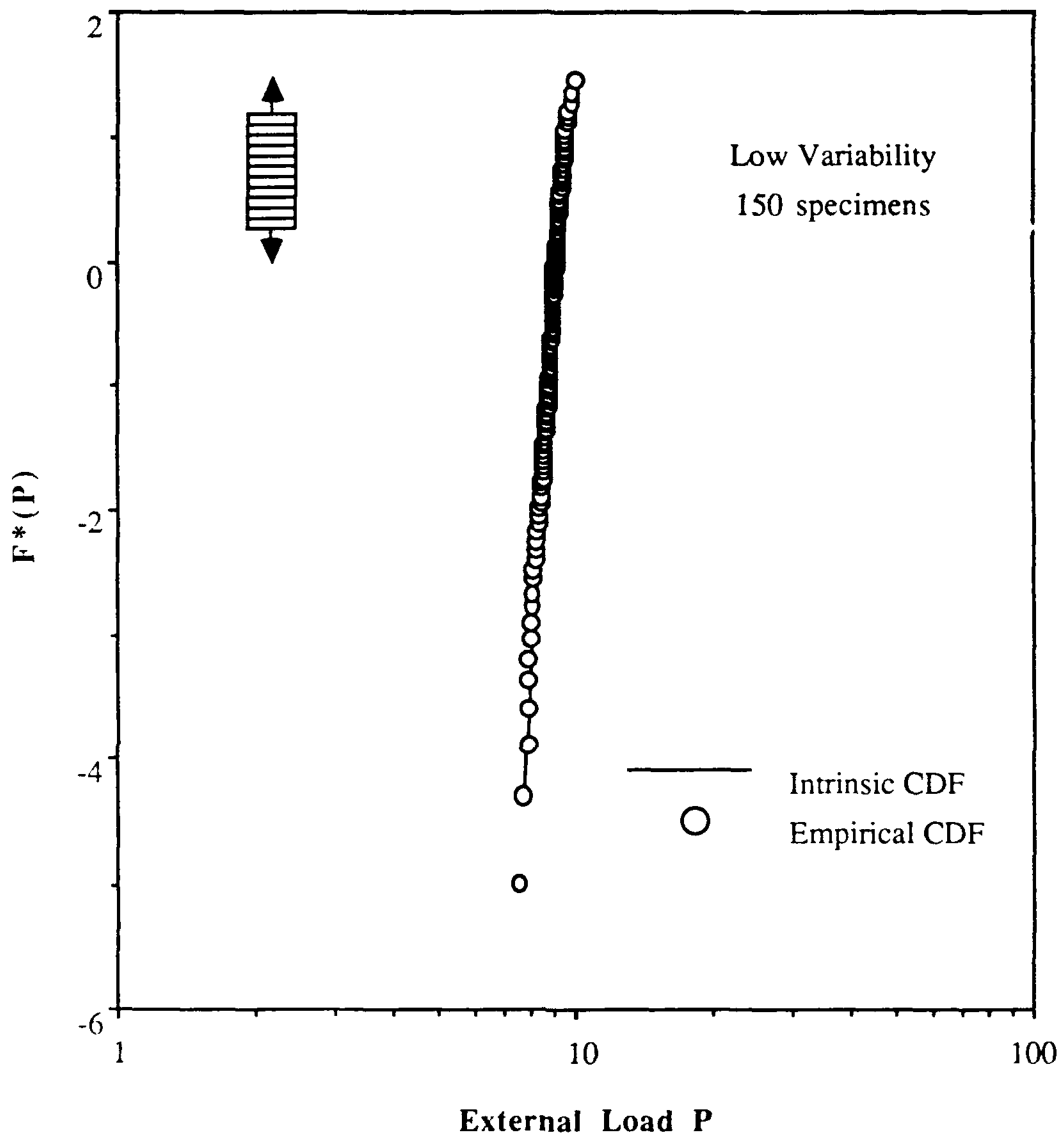


Figure 3-10(b). Comparison of intrinsic and empirical CDF's for homogeneous stress and low strength variability after 150 simulations.

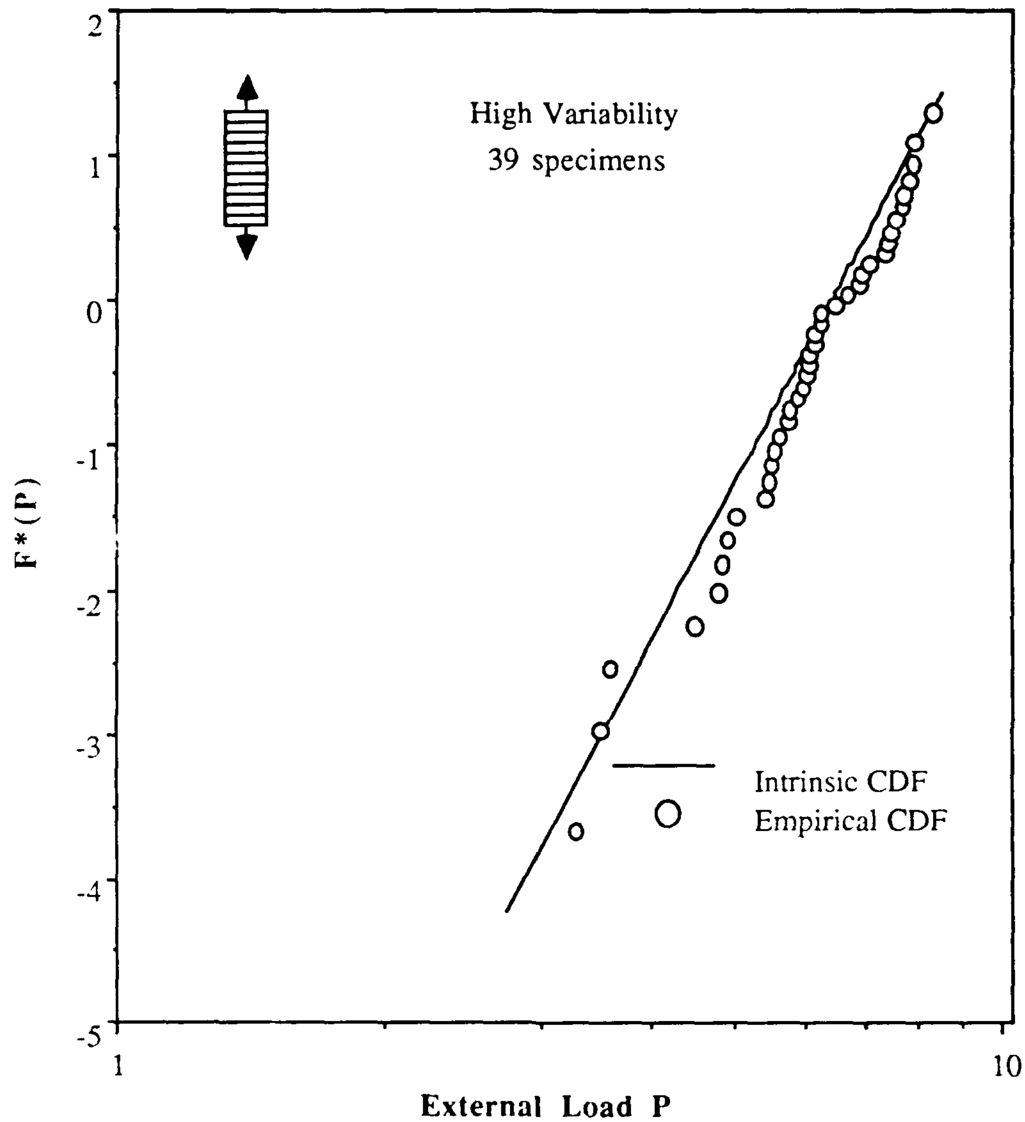


Figure 3-11(a). Comparison of intrinsic and empirical CDF's for homogeneous stress and high strength variability after 39 simulations.

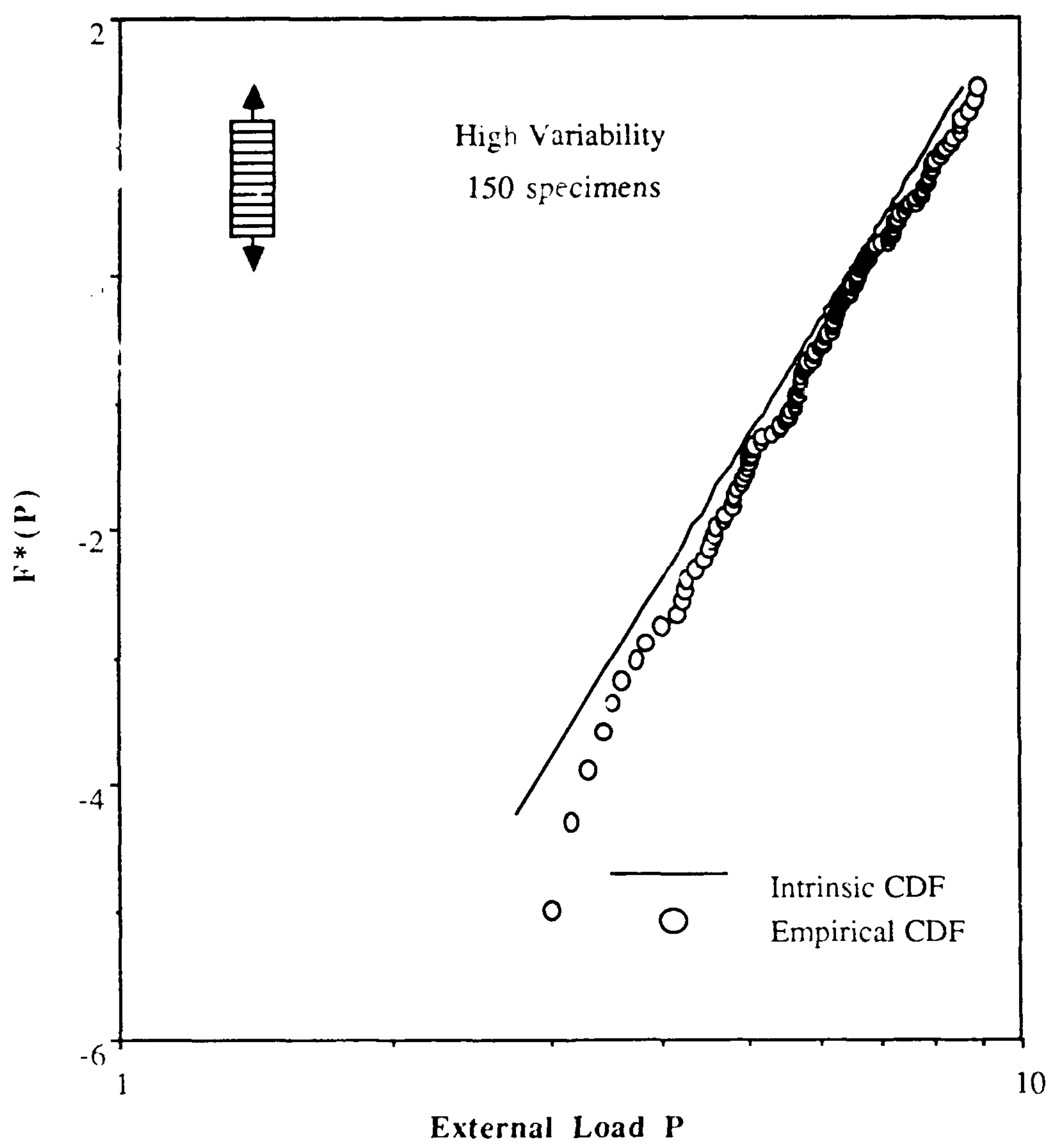


Figure 3-11(b). Comparison of intrinsic and empirical CDF's for homogeneous stress and high strength variability after 150 simulations.

2. Effect of Heterogeneous Stress Distribution on Failure Behavior

In a homogeneous stress distribution, the element with the lowest strength will be the source of structural failure regardless of its location. If the stress distribution is heterogeneous, the influence of the lowest elemental strength on the structural strength may be diminished if the weakest element has a low internal stress. Thus, the heterogeneity of the internal stress effectively increases the scale parameter of the empirical strength CDF for the structure. This is shown in Figures 3-12(a) and 3-12(b). The strengthening effect of the heterogeneous stress distribution is dependent on the intrinsic strength variability. As the variability decreases, the elemental strengths approach a uniform deterministic value at which magnitude of the statistical structural strength is maximum.

3. Effect of Structural Redundancy on Failure Behavior

The two-dimensional elemental distribution with heterogeneous stress distribution is compared with the one-dimensional elemental distribution with the same heterogeneous stress distribution to elucidate the effects of structural redundancy: local load sharing and the width size effect. The two effects can be visualized for high and low strength variability by using equal scale parameters for the intrinsic strength distribution (Figures 3-13(a) and 3-13(b)). In each case, the scale parameter of the empirical strength CDF for the structure appears to have decreased. In addition, the strength variability and failure probability appear to decrease at low failure loads. Since the internal stress distribution and, hence, the structural geometry of the two models are identical, the failure characterization of each should also be identical. This indicates that the strength statistics are dependent on the elemental width unless these effects are accounted for. If these

effects are taken into account, the parameters of the structural strength distribution will be analytically valid for any elemental distribution, but the scale parameter will still depend on the internal stress distribution.

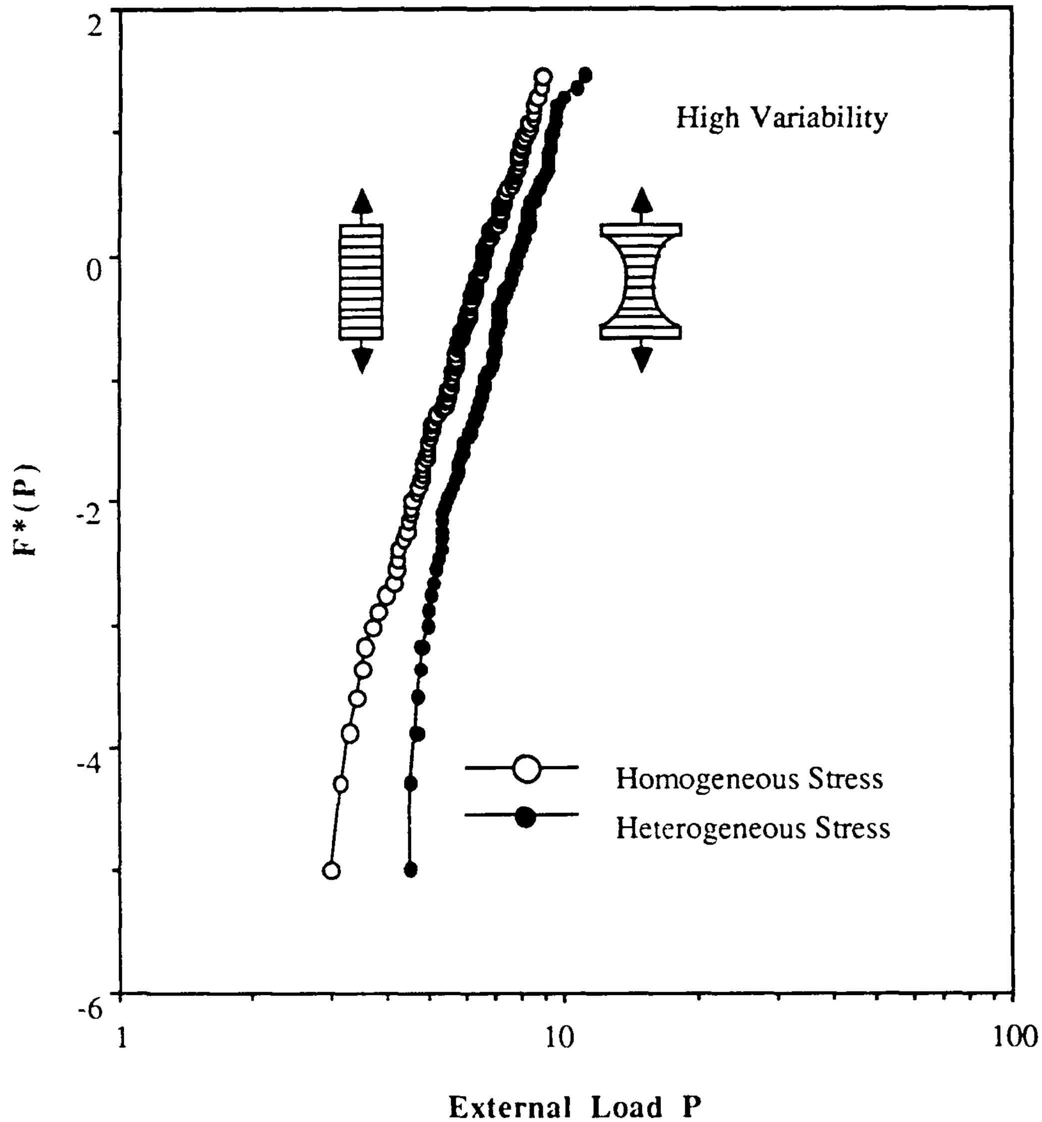


Figure 3-12(a). Comparison of empirical CDFs for homogeneous and heterogeneous stress distributions and high strength variability.

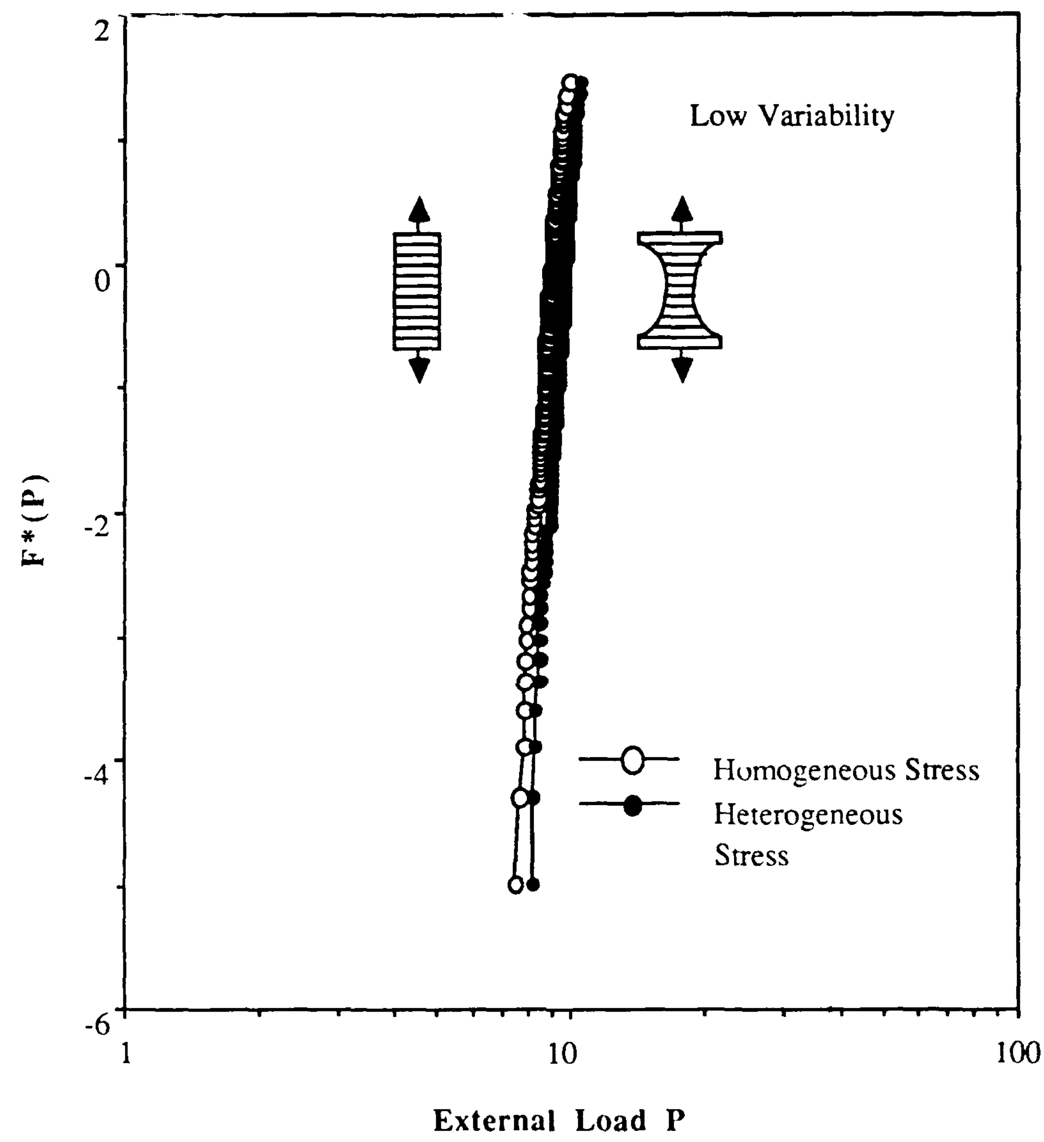


Figure 3-12(b). Comparison of empirical CDF's for homogeneous and heterogeneous stress distributions and low strength variability.

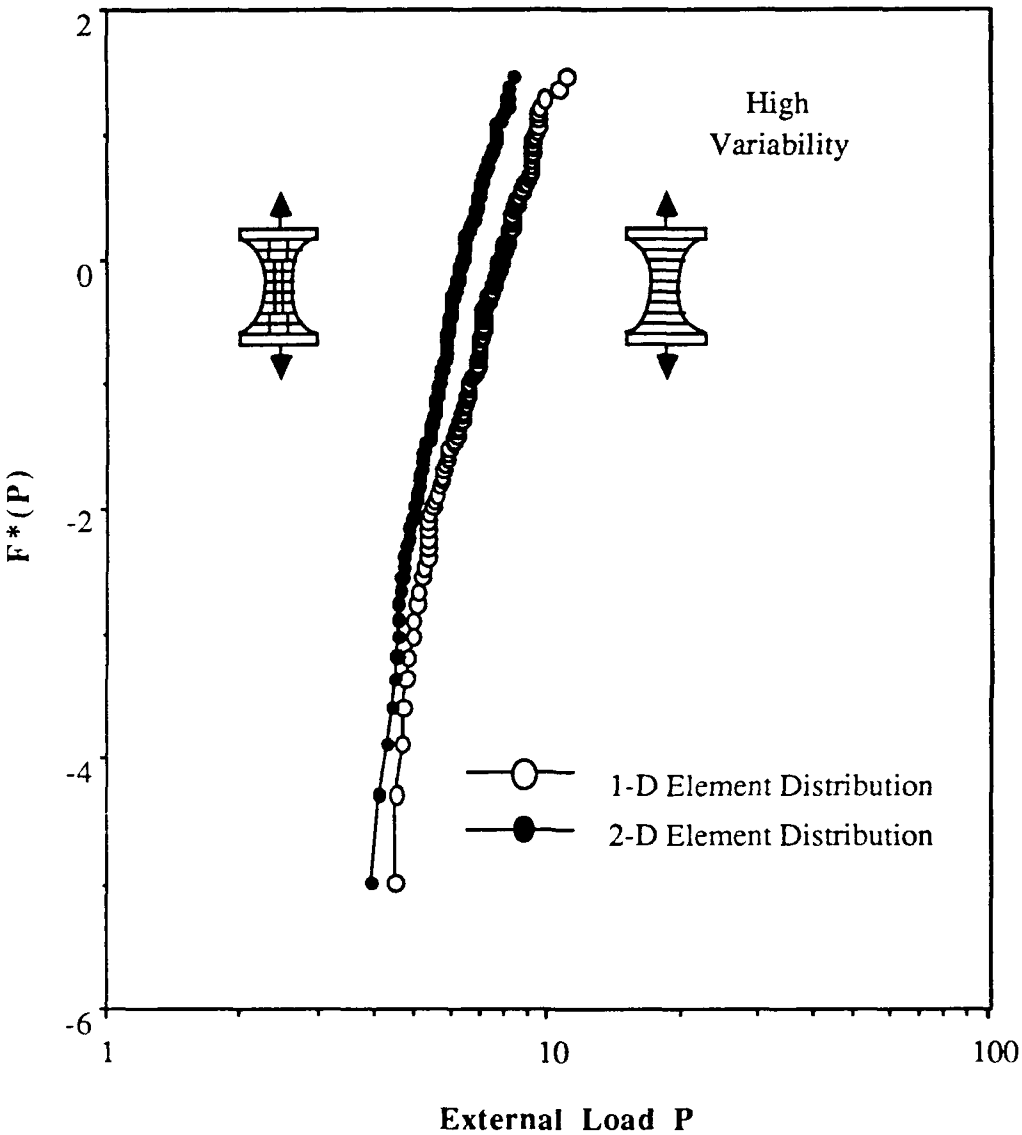


Figure 3-13(a). Comparison of empirical CDF's for one and two dimensional elemental distributions with heterogeneous stress and high strength variability.

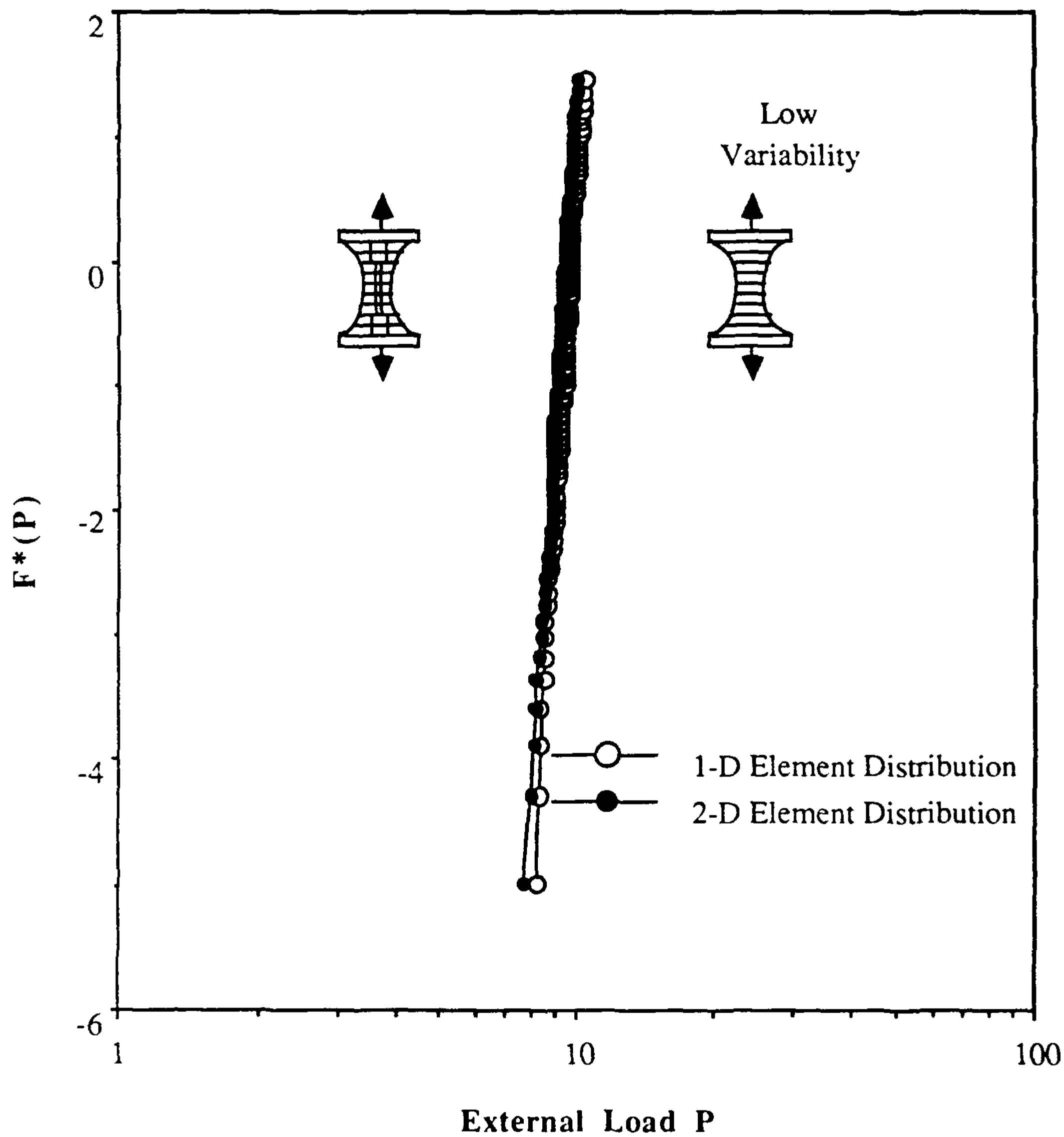


Figure 3-13(b). Comparison of empirical CDF's for one and two dimensional elemental distributions with heterogeneous stress and low strength variability.

The decrease in the scale parameter of the empirical CDF is the result of the width size effect. The two-dimensional elemental distribution gives rise to sequential failure of the structure which begins as the weak elements in the bundle fail. Since the internal stress distribution is heterogeneous, elemental failure is governed by the elemental stress-to-strength ratio, but bundle failure is dictated by the distribution of elemental intrinsic strengths within each bundle. As the number of elements in the bundle increases, the likelihood of very weak elements within the bundle increases and the sequential failure process within the bundle begins at a lower external load. This is the width size effect.

The decrease in strength variability and failure probability at low failure loads is caused by local load sharing within each bundle. Catastrophic failure of the structure tends to initiate once adjacent elements within a bundle have failed. Local load sharing tends to accelerate failure in elements adjacent to elements which have already have failed. A single element with a very high intrinsic strength can carry the additional load transferred to it by an adjacent failure sequence if that external load is relatively low. As the external load increases, even a very strong element within the bundle is unable to bear the transferred load and catastrophic structural failure ensues.

The width size effect and local load sharing effect appear to be dependent on strength variability. As the strength variability decreases, the range of elemental intrinsic strengths within a given bundle is limited and the strengths approach a uniform deterministic value. The adjacent, relatively weak elements required to start a failure sequence at low external load, and the relatively strong elements required to stop a failure sequence once it starts are not likely to be present. The two effects are diminished and failure characterization for low

strength variability in this case approaches that for the one-dimensional elemental distribution.

4. Geometrically Complex Models

The failure characterization for high strength variability of the plate model with a circular hole is compared with the "dogbone" model having a two-dimensional elemental distribution and heterogeneous stress distribution to illustrate the effect of a complex multiaxial stress distribution on one-dimensional failure characterization. Figure 3-14 compares the empirical structural strength distributions. For complex geometries such as the plate, determination of the mathematical transformation between failure load and failure stress is a complicated matter and requires numerical stress analysis.

All of the effects previously discussed are present in this comparison: length size effect, heterogeneous stress effect, width size effect and local load sharing effect. If all of these effects are considered and corrections included in the simulation, the resulting structural strength distributions should coincide.

Since the simulation was based on a uniaxial failure criterion, only a one-dimensional characterization of structural failure was possible for the plate; the contributions of transverse normal stress and shear stress to the structural failure characterization was not possible. In order to adequately model and numerically simulate probabilistic composite failure resulting from a combined stress distribution, a probabilistic failure criterion for combined stress is required. The formulation of such a failure criterion will be performed in the following chapter.

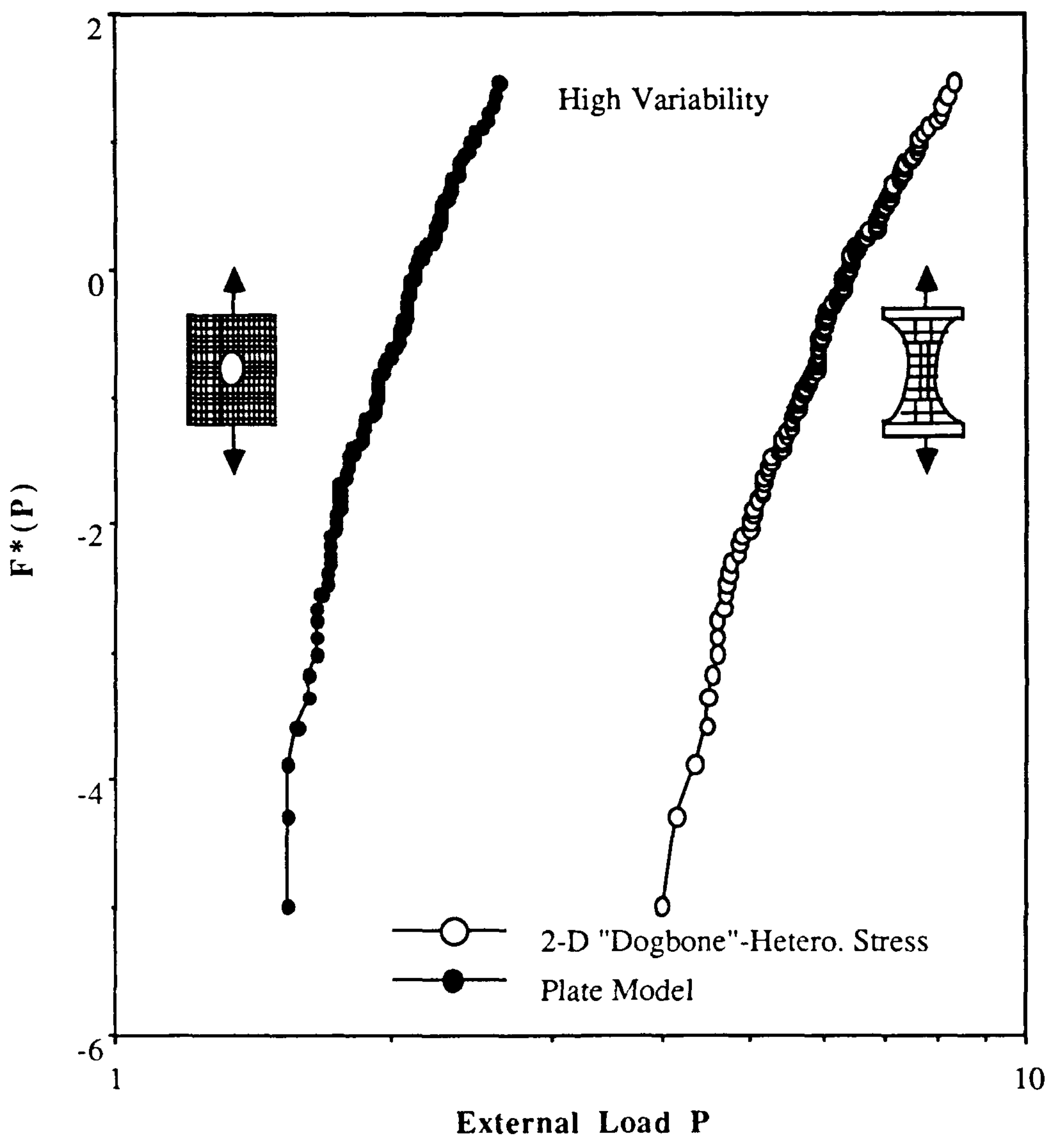


Figure 3-14. Comparison of empirical CDF's for the plate model and "dogbone" specimen with two dimensional elemental distribution.

IV. FORMULATION OF COMBINED STRESS FAILURE CRITERION

For a composite specimen, the applied stress tensor and the intrinsic strengths associated with each component of the applied stress tensor are represented as vectors in the combined stress space. The random intrinsic strength vectors, associated (colinear) with the applied stress vector, are manifest in failure modes tracible to the failure modes associated with failure under uniaxial stresses (i.e., the stress components of the vector). A probabilistic failure criterion for combined stress must account for each of these unique failure modes in terms of information which is experimentally or analytically available *a priori*, such as the micromechanical material behavior, the uniaxial intrinsic strength distributions and the applied stress tensor. This may be accomplished by determining the joint probability density function (pdf) in the stress space and integrating it over the domains associated with each failure mode. The failure modes may be either mechanistically and/or probabilistically interdependent (coupled) giving rise to four possible combinations of mechanistic coupling and probabilistic dependence. Excluding the case in which the failure modes are mechanistically and probabilistically coupled, the remaining three combinations would be experimentally indistinguishable and it is hypothesized in this investigation that a failure criterion formulated for one combination would be phenomenologically equivalent for all three combinations. Using the combination of mechanistically deterministic coupling and probabilistic independence, of the intrinsic strength the reliability of the composite may be expressed for the physical combined stress space in terms of Boolean operators. When the random vector functions defining

the failure modes in the combined stress space are substituted into the Boolean expression for reliability, the resulting transformation of random variables makes the random vector functions orthogonal and eliminates the coupling of the failure modes in the transformed space. The reliability in the transformed stress space may be converted to reliability in the physical stress space and the joint pdf in the physical stress space determined. The formulation presented herein will be limited to spatially two-dimensional tensile stress.

A. BACKGROUND

A composite specimen under combined stress has random intrinsic strengths corresponding to each component of the applied stress tensor. These intrinsic strengths, as well as the applied stress tensor, can be represented as colinear vectors whose direction in the combined stress space is known as the loading path. Each component of the stress tensor is manifest in a unique failure mode which, in the case of normal stress components, is also dependent on the sign of the stress component since different failure modes arise from tensile and compressive normal stresses. The occurrences of each failure mode for a composite specimen may be visualized in the combined stress space through thought experiments in which each intrinsic strength vector is observed over the range of all possible loading paths in the combined stress space.

In the case of biaxial tensile combined stress, the stress components and their associated intrinsic strengths may be represented as vectors with components along each axis of the biaxial stress space. The representation of a specimen under an arbitrary biaxial tensile stress is illustrated in Figure 4-1. Such a specimen under biaxial tensile combined stress has two possible failure mechanisms (modes), longitudinal (M_1) and transverse (M_2), each with an associated intrinsic strength

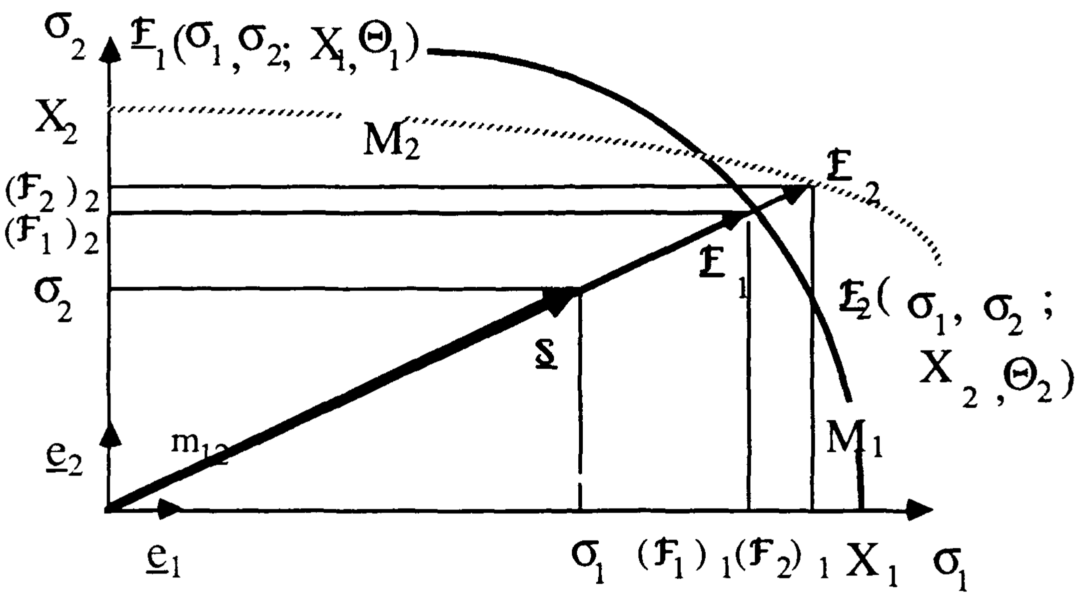


Figure 4-1. Representation of a specimen under arbitrary biaxial tensile stress.

vector, $\mathcal{F}_1$ and $\mathcal{F}_2$ respectively, which is colinear with the stress vector, $\mathcal{S}$. In the biaxial stress space, any loading path can be defined by m_{12}, which is the slope of the stress vector and its corresponding strength vectors. The failure mode which will occur in a given specimen along a particular loading path is determined by the smaller of the intrinsic strength vectors along the loading path, m_{12}.

Each specimen has associated with it random uniaxial intrinsic strengths, X_1 and X_2, along each axis of the biaxial stress space. In a thought experiment for an arbitrary specimen, each of the intrinsic strength vectors define a vector function in the biaxial stress space as m_{12} is varied over the range of the tensile domain, i.e., between zero and infinity. The function defined by the strength vector $\mathcal{F}_1$,

$$\mathcal{F}_1(\sigma_1,\sigma_2;X_1,\Theta_1) = (F_1)_1(\sigma_1,\sigma_2;X_1,\Theta_1)e_1 + \sigma_2 e_2 \tag{4-1}$$

intercepts the σ_1 axis at the uniaxial strength X_1, and has a set of coupling parameters given by Θ_1 which define its shape in the stress space in terms of the stress components, σ_1 and σ_2. Similarly, the curve defined by the strength vector $\underline{F}_2$,

$$\underline{F}_2\left(\sigma_1,\sigma_2;X_2,\Theta_2\right)=\sigma_1\underline{e}_1+\left(F_2\right)_2\left(\sigma_1,\sigma_2;X_2,\Theta_2\right)\underline{e}_2 \tag{4-2}$$

intercepts the σ_2 axis at the uniaxial strength X_2, and has a set of coupling parameters given by Θ_2. Each curve represents a deterministic failure criterion for a particular failure mode associated with that specimen; $\underline{F}_1(\sigma_1,\sigma_2;X_1,\Theta_1)$ represents failure mode M_1 and $\underline{F}_2(\sigma_1,\sigma_2;X_2,\Theta_2)$ represents failure mode M_2. The point of intersection of the two curves represents the stress state at which both failure modes occur simultaneously; this point is called the joint failure state. The location of the joint failure state with respect to the loading path determines the ultimate failure mode. This will be investigated in the following section.

B. GENERAL BOOLEAN REPRESENTATION OF FAILURE MODES

Once the physical conditions defining the occurrence of each possible failure mode for a composite subject to combined stress are defined mathematically, the probability of occurrence for each failure mode may be expressed in terms of logical (Boolean) operations. The magnitudes of the uniaxial intrinsic strength vectors are the random variables for these operations and the magnitude of the stress vector represents the realization of these random variables, i.e., the manifestation of the failure modes. When the mathematical representations for all

of the failure conditions are combined, a Boolean expression may be used to represent all of the possible failure conditions for a given loading path.

Failure results in mode M_1 for an arbitrary specimen in biaxial tensile stress is depicted in Figure 4-2. When the magnitude of the applied stress vector, $\underline{\mathbf{S}}$, equals or exceeds the magnitude of the intrinsic strength vector, $\underline{\mathfrak{F}}_1$, failure mode M_1 will result if the magnitude of $\underline{\mathfrak{F}}_1$ is the minimum of the strength vector magnitudes for the specimen. Thus, any specimen with a combination of intrinsic strength vectors, $\underline{\mathfrak{F}}_1$ and $\underline{\mathfrak{F}}_2$, such that $|\underline{\mathfrak{F}}_1| < |\underline{\mathfrak{F}}_2|$ will result in failure due to mode M_1 whenever $|\underline{\mathfrak{F}}_1| \leq |\underline{\mathbf{S}}|$. The probability of failure due to mode M_1 is:

$$F_c(|\underline{\mathbf{S}}|) = \Pr\{|\underline{\mathfrak{F}}_1| \leq |\underline{\mathbf{S}}|\} \tag{4-3}$$

given the condition that $|\underline{\mathfrak{F}}_1| < |\underline{\mathfrak{F}}_2|$. This condition is equivalent to the geometric configuration in which the joint failure states lie above the loading path.

Failure results in mode M_2 for a composite specimen is shown in Figure 4-3. When the magnitude of the applied stress vector, $\underline{\mathbf{S}}$, equals or exceeds the magnitude of the intrinsic strength vector, $\underline{\mathfrak{F}}_2$, failure mode M_2 will occur if the magnitude of $\underline{\mathfrak{F}}_2$ is the minimum of the strength vector magnitudes for the specimen. In this instance, any specimen with a combination of intrinsic strength vectors, $\underline{\mathfrak{F}}_1$ and $\underline{\mathfrak{F}}_2$, such that $|\underline{\mathfrak{F}}_2| < |\underline{\mathfrak{F}}_1|$ will result in failure due to mode M_2 when $|\underline{\mathfrak{F}}_2| \leq |\underline{\mathbf{S}}|$. Failure due to mode M_2 may therefore be represented in Boolean notation as

$$F_c(|\underline{\mathbf{S}}|) = \Pr\{|\underline{\mathfrak{F}}_2| \leq |\underline{\mathbf{S}}|\} \tag{4-4}$$

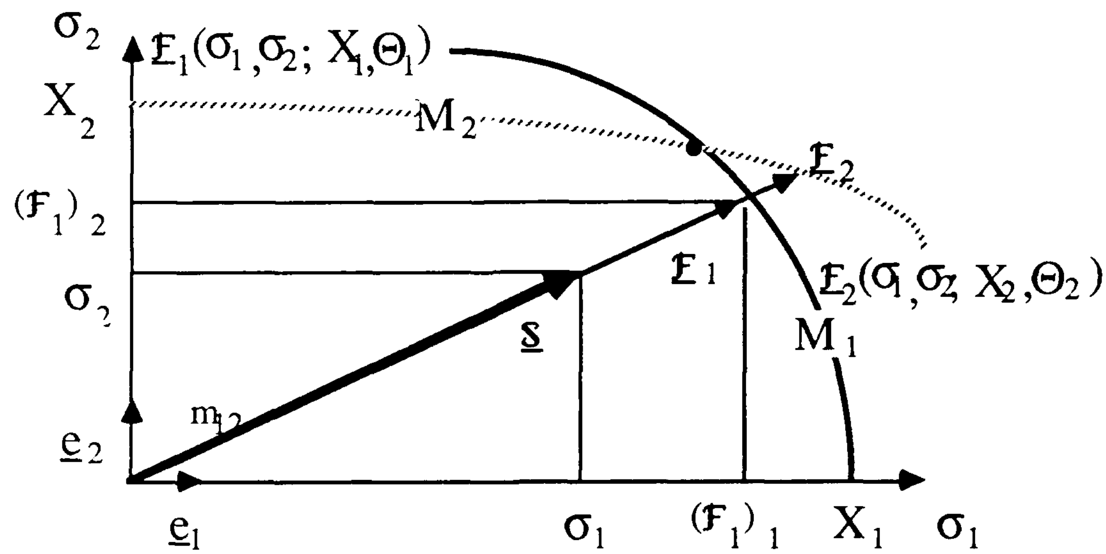


Figure 4-2. Representation of failure due to mode M. in biaxial stress space.

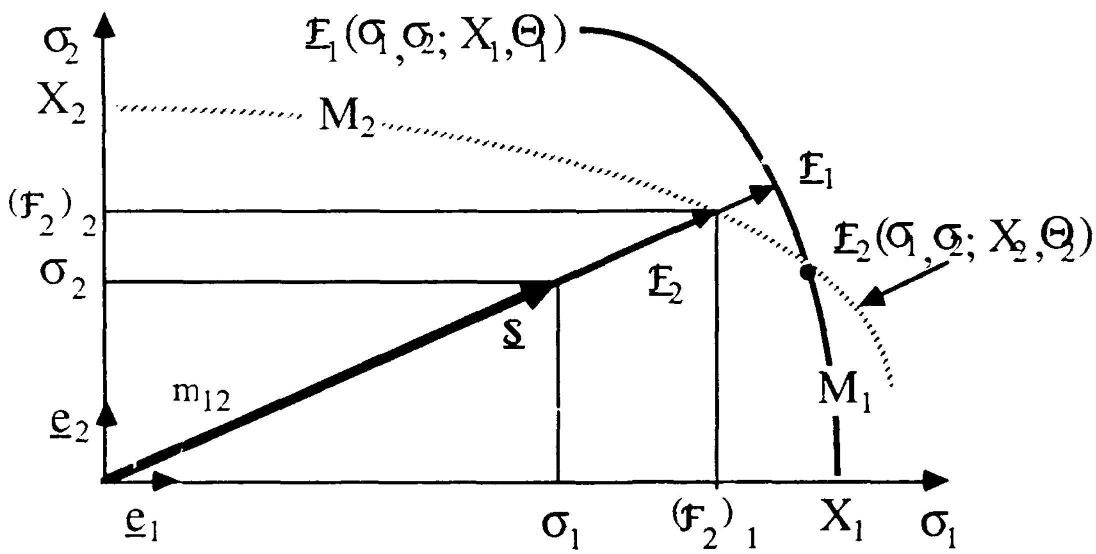


Figure 4-3. Representation of failure due to mode M_2 in biaxial stress space.

given that $|\underline{\mathcal{F}}_2| < |\underline{\mathcal{F}}_1|$. Similarly, this condition is equivalent to the geometric configuration in which the joint failure states lie below the loading path.

By combining the failure conditions for each failure mode, the failure condition for the specimen may be expressed in terms of logical statements. Failure occurs when the magnitude of the stress vector equals or exceeds the minimum magnitude of the intrinsic strength vectors or,

$$|\underline{S}| \geq |\underline{\mathcal{F}}_1| \wedge |\underline{\mathcal{F}}_2| \tag{4-5}$$

The probability of failure for either failure mode may now be expressed as

$$F_c(|\underline{S}|) = \Pr\{|\underline{\mathcal{F}}_1| \wedge |\underline{\mathcal{F}}_2| \leq |\underline{S}|\} \tag{4-6}$$

The requirement for the minimum strength magnitude may be eliminated if Equation (4-6) is recast in terms of the composite reliability. While composite failure is defined as the occurrence of either failure mode, composite reliability is defined as the occurrence of neither failure mode and is defined as

$$R_c(|\underline{S}|) = 1 - F_c(|\underline{S}|) \tag{4-7}$$

or, in terms of the strength vector magnitudes,

$$R_c(|\underline{S}|) = \Pr\{|\underline{\mathcal{F}}_1| \wedge |\underline{\mathcal{F}}_2| > |\underline{S}|\} \tag{4-8}$$

Equation (4-8) implies that the magnitudes of each strength vector must be greater than the magnitude of the stress vector. Thus, two conditions must both

apply for the composite to be reliable, $|\underline{\Sigma}_1| > |\underline{S}|$ and $|\underline{\Sigma}_2| > |\underline{S}|$. The composite reliability may now be expressed in terms of these two conditions

$$R_c(|\underline{S}|) = \Pr\{[|\underline{\Sigma}_1| > |\underline{S}|] \text{ and } [|\underline{\Sigma}_2| > |\underline{S}|]\} \tag{4-9}$$

In terms of Boolean operations, Equation (4-9) represents the intersection of the two reliability conditions and may be rewritten as

$$R_c(|\underline{S}|) = \Pr\{[|\underline{\Sigma}_1| > |\underline{S}|] \cap [|\underline{\Sigma}_2| > |\underline{S}|]\} \tag{4-10}$$

Equation (4-10) is a general expression for the reliability of a composite under biaxial tensile stress conditions. However, additional simplifications must still be made before it may be effectively applied. These simplifications will be identified in subsequent sections of this investigation .

C. COUPLING OF FAILURE MODES

When the deterministic failure criteria which represent the failure modes of a single specimen in the biaxial stress space are orthogonal to one another, the failure modes of the composite are considered independent (uncoupled). These conditions are schematically illustrated in Figure 4-4. Conversely, deviation from orthogonality by the specimen failure criteria indicates that the failure modes are coupled. Coupling may result from the effect of the orthogonal stress components on the uniaxial intrinsic strength associated with the other stress component. This is called mechanistic coupling and is governed by the coupling parameters, Θ. Coupling may also result from an interdependency between the statistical ordering of the random intrinsic strength vector components. This condition is defined as

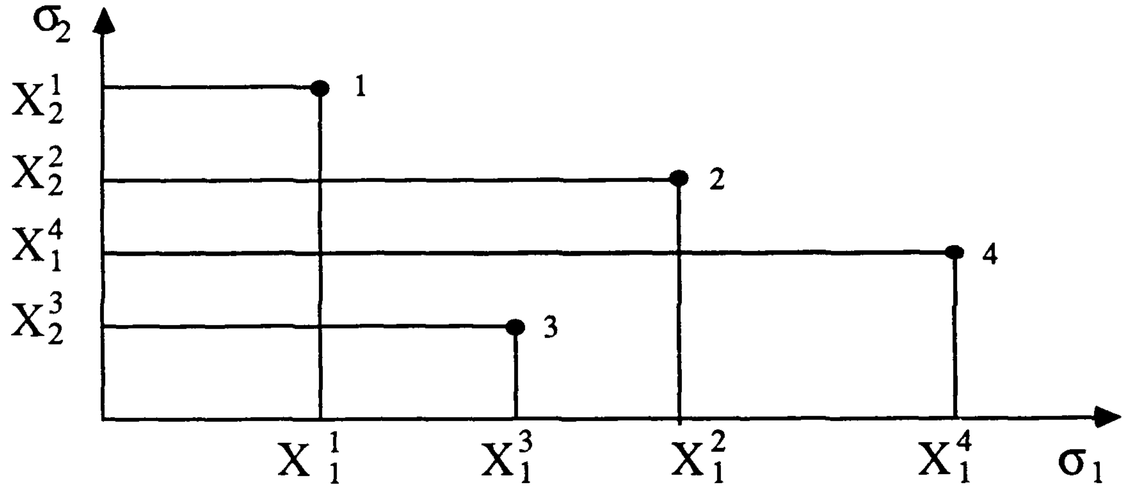


Figure 4-4. Independent (uncoupled) failure modes in the biaxial stress space.

probabilistic coupling and is the result of an interdependency between the relative statistical orders of the random uniaxial intrinsic strengths.

1. **Mechanistic Coupling**

The specimen failure criteria are represented in the biaxial stress space as $\mathcal{F}_1(\sigma_1,\sigma_2;X_1,\Theta_1)$ and $\mathcal{F}_2(\sigma_1,\sigma_2;X_2,\Theta_2)$. The stress component dependencies of these failure criteria are determined by the coupling parameters, Θ_1 and Θ_2. If Θ_1 and Θ_2 are constant parameters, then both $\mathcal{F}_1(\sigma_1,\sigma_2;X_1,\Theta_1)$ and $\mathcal{F}_2(\sigma_1,\sigma_2;X_2,\Theta_2)$ will be homologous; i.e., each will maintain the same shape in the biaxial stress space for every specimen. This is defined as mechanistically deterministic coupling and is schematically illustrated in Figure 4-5(a). If Θ_1 and Θ_2 are random parameters for each specimen, then each failure criterion will have different stress dependencies, and therefore different shapes, in the biaxial stress space for different specimens. This is defined as mechanistically probabilistic and is illustrated in Figure 4-5(b). In the latter case, the failure criteria would have the

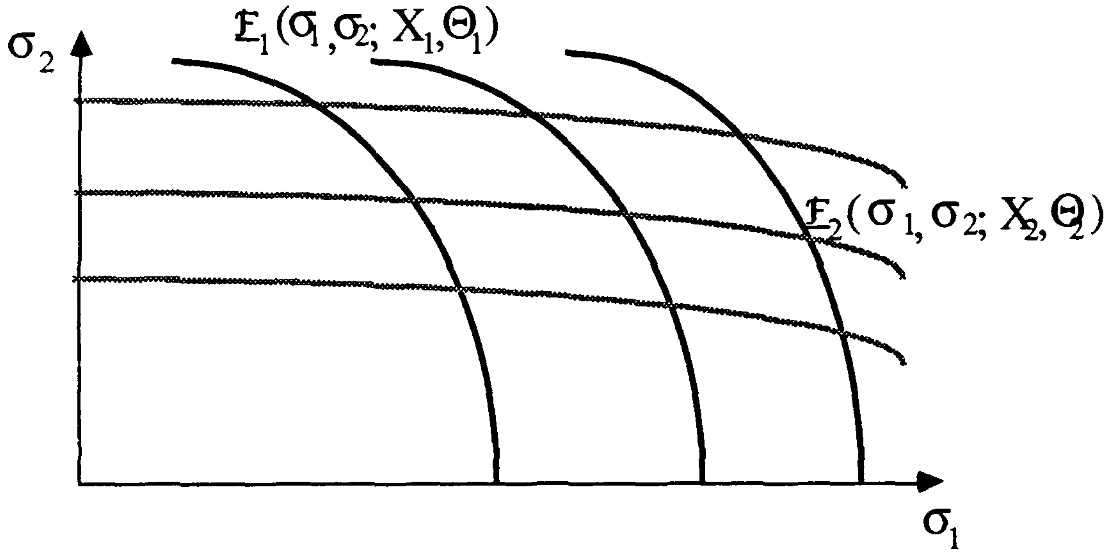


Figure 4-5(a). Mechanistically deterministic coupling.

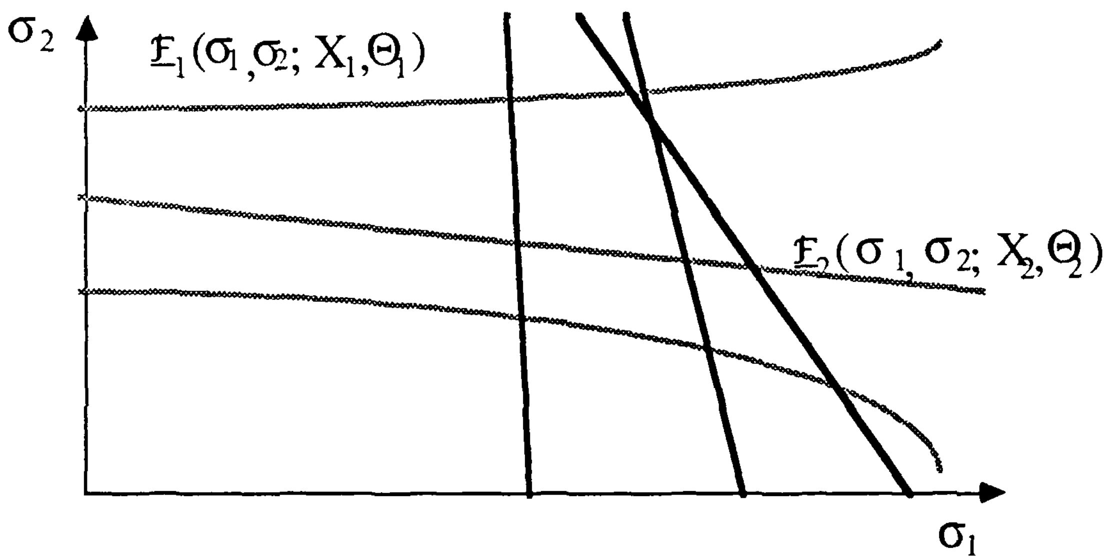


Figure 4-5(b). Mechanistically probabilistic coupling.

functional forms $\mathfrak{F}_1(\sigma_1,\sigma_2; X_1,\Theta_1(\sigma_1))$ and $\mathfrak{F}_2(\sigma_1,\sigma_2; X_2,\Theta_2(\sigma_2))$ where $\Theta_1(\sigma_1)$ and $\Theta_2(\sigma_2)$ are represented by statistical distribution functions.

2. Probabilistic Coupling

The components of the random intrinsic strength vectors may be dependent on one another if their associated random uniaxial intrinsic strengths are interdependent. An example of intrinsic dependency may be visualized in the pairing of the uniaxial intrinsic strengths by the rank ordering of their magnitudes (i.e., low strength to low strength, high strength to high strength, etc.). The rank order of the random uniaxial intrinsic strengths for the population of specimens are determined by their respective statistical distribution functions, $F_{X_1}(\sigma_1)$ and $F_{X_2}(\sigma_2)$, shown in Figure 4-6 and the mechanistically deterministic coupling in Figure 4-7. The failure modes occurring along entire segments of a given loading path will be the same if the coupling parameters of the failure criteria are deterministic and will vary randomly if the coupling parameters are probabilistic.

On the other hand, if the components of the random intrinsic strength vectors are independent of one another, the uniaxial intrinsic strengths for any specimen will be randomly paired. The failure modes occurring along any loading path will vary randomly regardless of the nature of the coupling parameters. The spatial distribution of joint failure states in the three combinations of mechanistic and probabilistic coupling where the failure modes vary randomly along any loading path are apparently indistinguishable from one another. Therefore, the same mathematical formulation of the probabilistic failure criterion for the composite should be valid for any of the three combinations of coupling.

For combined biaxial stress, there may exist an interdependency between the uniaxial intrinsic strengths, X_1 and X_2. If so, then the components of the failure

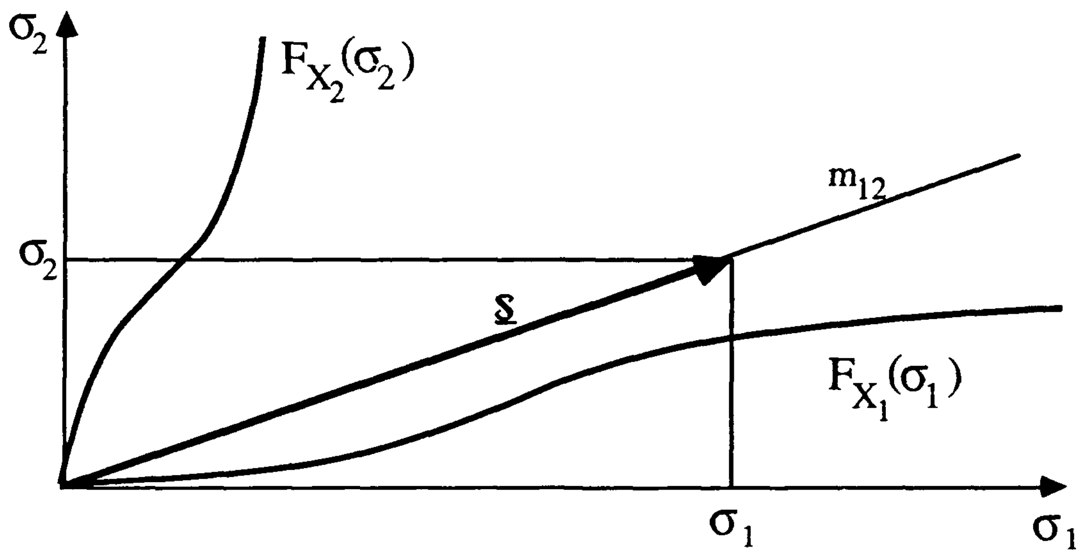


Figure 4-6. Uniaxial strength distribution functions in the biaxial stress space.

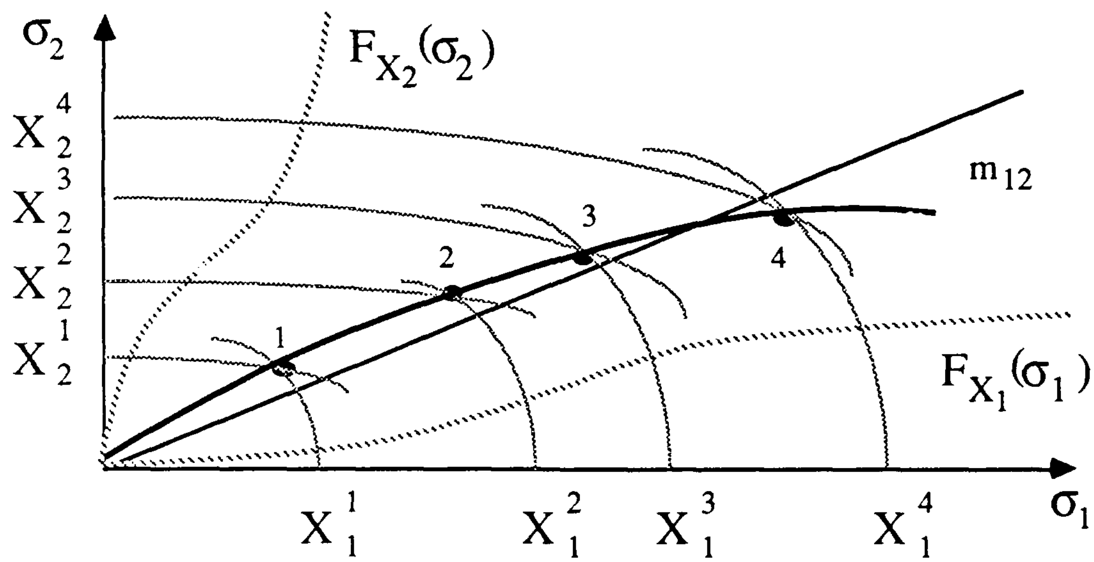


Figure 4-7. Mechanistically deterministic and probabilistically dependent coupling.

criteria, $(\mathcal{F}_1)_1\ (\sigma_1,\sigma_2;X_1,\Theta_1)$and $(\mathcal{F}_2)_2(\sigma_1,\sigma_2;X_2,\Theta_2)$, the rank order of which are determined by the respective distributions, $F_{(\mathcal{F}_1)_1}(\sigma_1,\sigma_2;X_1,\Theta_1)$ and $F_{(\mathcal{F}_2)_2}(\sigma_1,\sigma_2;X_2,\Theta_2)$, would also be interdependent and the failure modes are considered probabilistically dependent. In this case, a specimen with a low uniaxial intrinsic strength in the longitudinal direction will have a low uniaxial intrinsic strength in the transverse direction; a specimen with a high uniaxial intrinsic strength in the longitudinal direction will have a high uniaxial intrinsic strength in the transverse direction. In terms of the distributions of the failure criteria components,

$$F_{(\mathcal{F}_1)_1}\left(\sigma_1,\sigma_2=0;X_1,\Theta_1\right)=F_{(\mathcal{F}_2)_2}\left(\sigma_1=0,\sigma_2;X_2,\Theta_2\right) \tag{4-12}$$

If, in a thought experiment for this case, the uniaxial intrinsic strengths for all specimens in the population were rank ordered and their individual failure criteria plotted in the biaxial stress space, a continuous locus of joint failure states for the population may also be plotted. Given any arbitrary loading path, the failure modes occurring over entire segments along the loading path are the same. If the uniaxial strength distributions have the same shape parameter, then the failure modes occurring along a given loading path remain the same along the entire path. These phenomena may be visualized in Figures 4-8(a) and 4-8(b).

The occurrences of the failure modes are different in the case where no interdependency exists between the uniaxial intrinsic strengths, X_1 and X_2. The pairing of the uniaxial strengths is random; and the failure modes are considered probabilistically independent. In this case, a specimen with a low uniaxial intrinsic

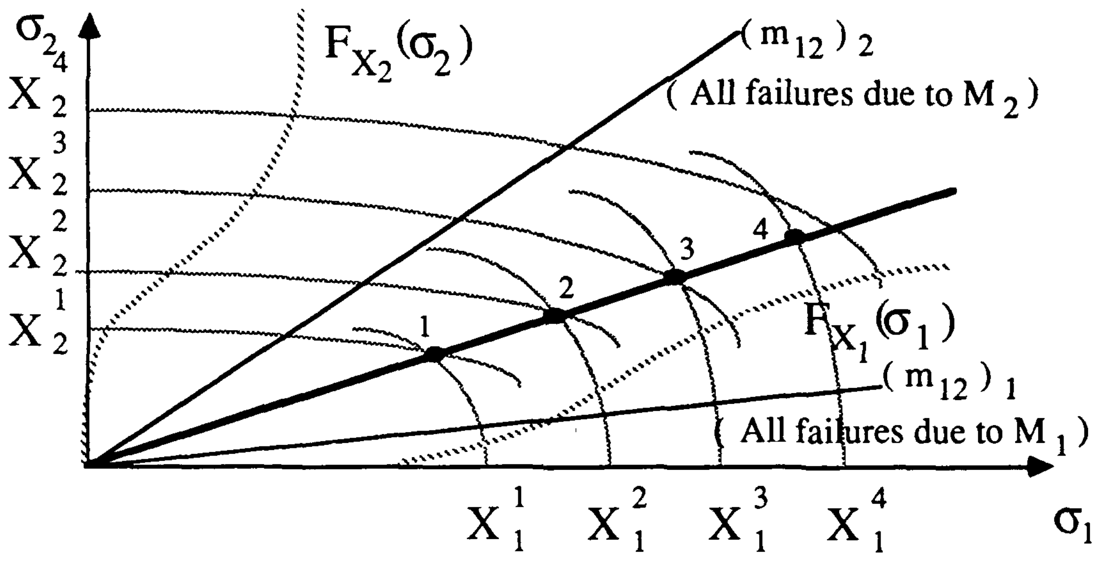


Figure 4-8(a). Mechanistically deterministic, probabilistically dependent coupling with uniaxial strength distributions having equal shape parameters.

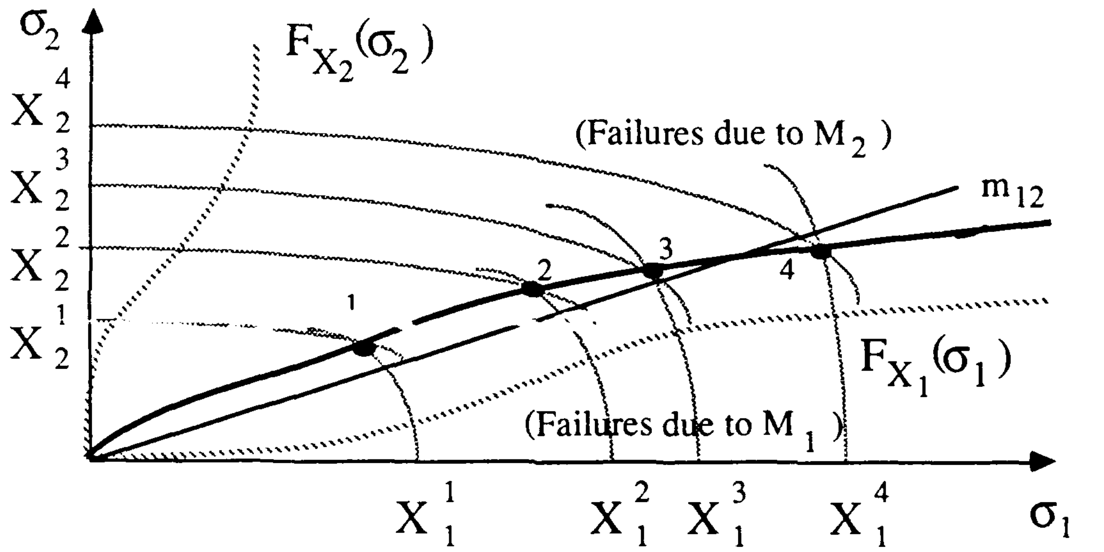


Figure 4-8(b). Mechanistically deterministic, probabilistically dependent coupling with uniaxial strength distributions having unequal shape parameters.

strength in the longitudinal direction may have a high uniaxial intrinsic strength in the transverse direction, etc.

There are four possible combinations of mechanistic and probabilistic coupling in a population of composite specimens. The combination of deterministic mechanisms and probabilistic dependence has unique characteristics and has been previously examined. The remaining three combinations, deterministic mechanisms with probabilistic independence, probabilistic mechanisms with probabilistic dependence and probabilistic mechanisms with probabilistic independence, share one significant characteristic. In each case, no continuous locus of joint failure states may be plotted in the biaxial stress space. Hence, a random mixture of failure modes will result along any given loading path. This phenomenon is illustrated in Figures 4-9(a), 4-9(b) and 4-9(c).

Depending on the coupling parameters associated with the failure modes, the failure characterizations of the three combinations may be indistinguishable in the biaxial stress space. For this reason, the cumulative combined-stress failure criterion for one combination will be assumed, for the purposes of this investigation, to apply to all three combinations. In the following section the combined-stress probabilistic failure criterion will be formulated for the combination of deterministically coupled failure modes with probabilistic independence.

D. FORMULATION OF FAILURE CRITERION

The joint failure probability density function (pdf) for the biaxial stress space is defined as the probability that both failure modes M_1 and M_2 occur at any given biaxial stress state in the composite and is expressed in terms of the applied stress components. If the joint failure pdf for the biaxial stress space and the spatial

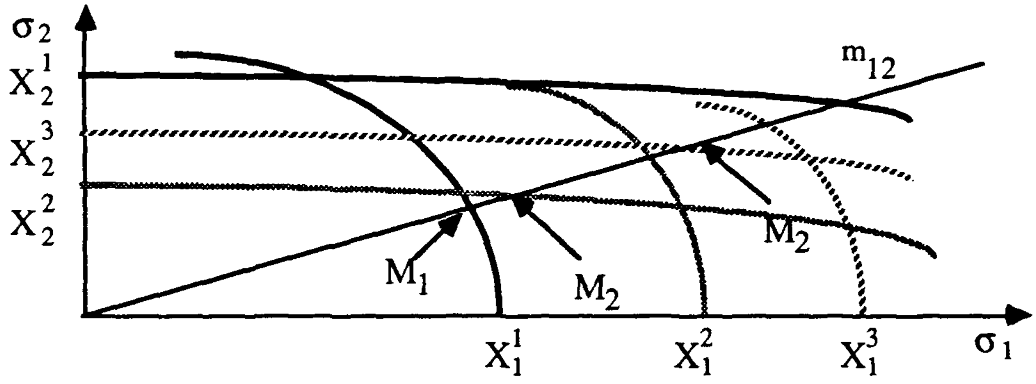


Figure 4-9(a). Mechanistically deterministic, probabilistically independent coupling.

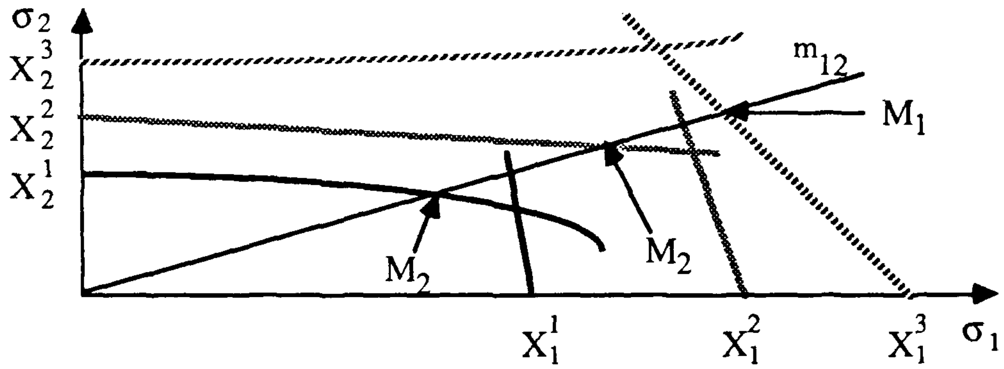


Figure 4-9(b). Mechanistically probabilistic, probabilistically dependent coupling.

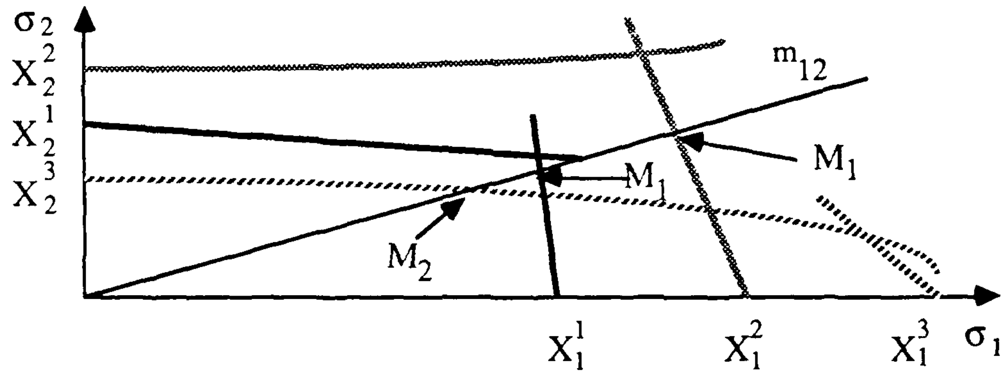


Figure 4-9(c). Mechanistically probabilistic, probabilistically independent coupling.

domains associated with each failure mode are known, then the probability of failure for the composite may be obtained by integrating the joint failure pdf over the stress domains for both failure modes. However, the joint failure pdf is usually not available. In this case, the stress components may be mapped into a transformed stress space for which the random variables are independent and the probability of failure in the transformed space determined. The joint failure pdf for the physical stress space may then be determined by differentiating the transformed probability of failure with respect to the physical stress components. A mathematical model for the mechanistic dependency will be proposed based on the micromechanical behavior of composite failure processes. This mechanistic dependency model will then be used for transformation and derivation of the joint probability failure function.

1. Formulation for General Mechanistic Coupling

The objective of this formulation is to obtain the joint pdf for a composite in the tensile biaxial stress space for an arbitrary coupling of failure modes. Under the hypothesis that the three of the four combinations of mechanistic and probabilistic coupling are indistinguishable in the biaxial stress space, the formulation of a probabilistic failure criterion for one combination will be applicable to all three. The strength coupling case for mechanistically deterministic coupling in conjunction with probabilistically independent intrinsic strengths will be explored. Under these conditions, the coupling parameters of the specimen failure criteria are constant and that the random uniaxial intrinsic strengths are independent of one another.

The deterministic specimen failure criteria have been defined in vector form by Equations (4-1) and (4-2):

$$\mathfrak{F}_1(\sigma_1,\sigma_2;X_1,\Theta_1)=(\mathcal{F}_1)_1(\sigma_1,\sigma_2;X_1,\Theta_1)\underline{e}_1+\sigma_2\underline{e}_2$$

$$\mathfrak{F}_2(\sigma_1,\sigma_2;X_2,\Theta_2)=\sigma_1\underline{e}_1+(\mathcal{F}_2)_2(\sigma_1,\sigma_2;X_2,\Theta_2)\underline{e}_2$$

The failure criterion for each failure mode can be partitioned into the uniaxial strength, X_i, plus a coupling effect, $C_{ij}(\sigma_j)$:

$$(\mathcal{F}_1)_1(\sigma_1,\sigma_2;X_1,\Theta_1)=X_1+C_{12}(\sigma_2) \tag{4-13}$$

and

$$(\mathcal{F}_2)_2(\sigma_1,\sigma_2;X_2,\Theta_2)=X_2+C_{21}(\sigma_1) \tag{4-14}$$

where $C_{12}(\sigma_2)$ and $C_{21}(\sigma_1)$ represent arbitrary coupling functions. The uniaxial failure pdf's of the stress components are taken to be known and denoted respectively by $f_{X_1}(\sigma_1)$ and $f_{X_2}(\sigma_2)$.

The general Boolean expression for composite reliability in vector form was previously given by Equation (4-10) stating the logical conditions that a sample is reliable only when the strength vectors for each mode both exceed the applied stress vector:

$$R_c(|\underline{S}|)=\Pr\{[|\mathfrak{F}_1|>|\underline{S}|]\cap[|\mathfrak{F}_2|>|\underline{S}|]\}$$

If the strength vector is assumed to be path independent, then Equation (4-10) may be expressed in terms of the scalar components, $(\mathcal{F}_1)_1$ or $(\mathcal{F}_1)_2$, and

$(\mathcal{F}_2)_1$ or $(\mathcal{F}_2)_2$. The reliability condition for failure mode M_1 can be expressed in terms of the uniaxial condition, $(\mathcal{F}_1)_1 > \sigma_1$ and the reliability condition for failure mode M_2 can be expressed in terms of its uniaxial condition, $(\mathcal{F}_2)_2 > \sigma_2$. Equation (4-10) may then be rewritten in terms of the strength vector components:

$$R_c(\sigma_1,\sigma_2) = \Pr\left\{\left[(\mathcal{F}_1)_1 > \sigma_1\right] \cap \left[(\mathcal{F}_2)_2 > \sigma_2\right]\right\} \tag{4-15}$$

Equation (4-15) is the intersection of the reliability conditions for the two failure modes and may be expressed in terms of the conditional probabilities of the two reliability conditions:

$$R_c(\sigma_1,\sigma_2) = \Pr\left\{\left[(\mathcal{F}_1)_1 > \sigma_1\right] \middle| \left[(\mathcal{F}_2)_2 > \sigma_2\right]\right\} \Pr\left\{(\mathcal{F}_2)_2 > \sigma_2\right\} \tag{4-16a}$$

or,

$$R_c(\sigma_1,\sigma_2) = \Pr\left\{\left[(\mathcal{F}_2)_2 > \sigma_2\right] \middle| \left[(\mathcal{F}_1)_1 > \sigma_1\right]\right\} \Pr\left\{(\mathcal{F}_1)_1 > \sigma_1\right\} \tag{4-16b}$$

The scalar components of the strength vectors, $\underline{\mathcal{F}}_1$ and $\underline{\mathcal{F}}_2$, $(\mathcal{F}_1)_1$ and $(\mathcal{F}_2)_2$, are related to the uniaxial strengths, X_1 and X_2, by Equations (4-13) and (4-14). Substituting these relations into Equations (4-16a) and (4-16b) and rearranging the inequalities in terms of X_1 and X_2,

$$R_c(\sigma_1,\sigma_2) =$$

$$\Pr\left\{\left\{X_1>\left[\sigma_1-C_{12}(\sigma_2)\right]\right\}\Big|\left\{X_2>\left[\sigma_2-C_{21}(\sigma_1)\right]\right\}\right\}\Pr\left\{X_2>\left[\sigma_2-C_{21}(\sigma_1)\right]\right\} \tag{4-17a}$$

or,

$$R_c(\sigma_1,\sigma_2)=$$

$$\Pr\left\{\left\{X_2>\left[\sigma_2-C_{21}(\sigma_1)\right]\right\}\Big|\left\{X_1>\left[\sigma_1-C_{12}(\sigma_2)\right]\right\}\right\}\Pr\left\{X_1>\left[\sigma_1-C_{12}(\sigma_2)\right]\right\} \tag{4-17b}$$

Rearranging the inequalities in terms of X_1 and X_2 is equivalent to transforming the random variables from the strength vector components to the uniaxial strengths. The composite reliability has been mapped from the physical stress space defined in terms of σ_1 and σ_2 into a transformed stress space defined in terms of transformed stress components σ_1' and σ_2' by

$$\sigma_1'=\sigma_1-C_{12}(\sigma_2) \tag{4-18a}$$

and,

$$\sigma_2'=\sigma_2-C_{21}(\sigma_1) \tag{4-18b}$$

Differentiating Equations (4-18a) and (4-18b) gives the relations

$$\frac{\partial\sigma_1'}{\partial\sigma_1}=\frac{\partial\sigma_2'}{\partial\sigma_2}=1 \tag{4-19a}$$

$$\frac{\partial\sigma_1'}{\partial\sigma_2}=\frac{dC_{12}}{d\sigma_2} \tag{4-19b}$$

$$\frac{\partial\sigma_2'}{\partial\sigma_1}=\frac{dC_{21}}{d\sigma_1} \tag{4-19c}$$

and,

$$\frac{\partial^2\sigma_1'}{\partial\sigma_1\partial\sigma_2}=\frac{\partial^2\sigma_2'}{\partial\sigma_1\partial\sigma_2}=0 \tag{4-19d}$$

Figure 4-10 depicts the transfonned stress space. Equations (4-17a) and (4-17b) may be rewritten in terms of $\sigma_1'$and σ_2' as

$$R_c(\sigma_1,\sigma_2)=\Pr\left\{[X_1>\sigma_1']\,|\,[X_2>\sigma_2']\right\}\Pr\{X_2>\sigma_2'\} \tag{4-20a}$$

and,

$$R_c(\sigma_1,\sigma_2)=\Pr\left\{[X_2>\sigma_2']\,|\,[X_1>\sigma_1']\right\}\Pr\{X_1>\sigma_1'\} \tag{4-20b}$$

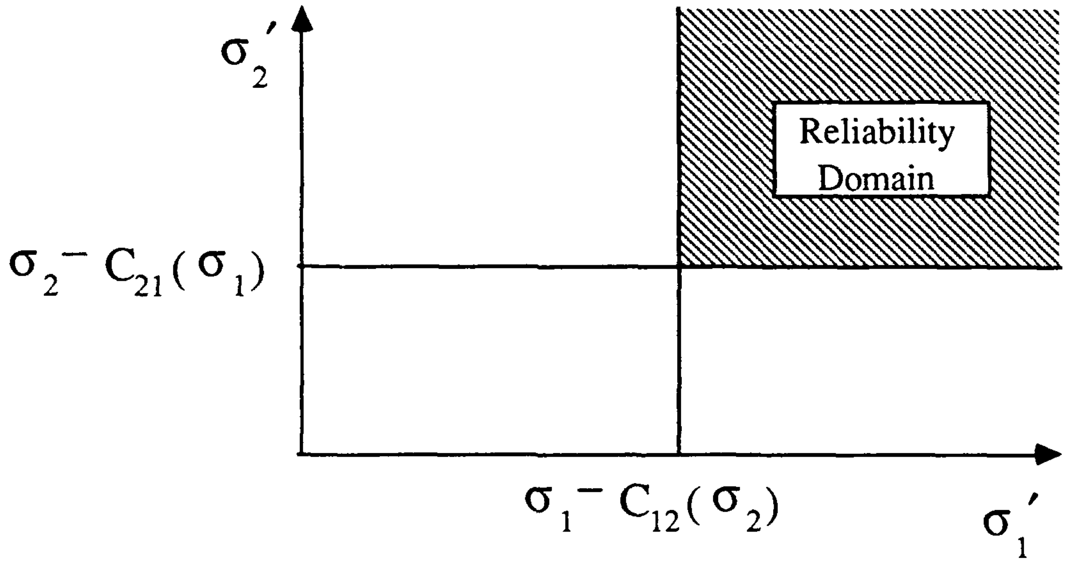


Figure 4-10...Transformed biaxial stress space.

The uniaxial strengths , X_1 and X_2, are taken as independent random variables. Therefore, in terms of the transformed stress components, $\sigma_1'$and σ_2',the conditional probabilities in Equations (4-20a) and (4-20b) become

$$\Pr\left\{\left[X_1>\sigma_1'\right]\middle|\left[X_2>\sigma_2'\right]\right\}=\Pr\left\{X_1>\sigma_1'\right\} \tag{4-21}$$

and,

$$\Pr\left\{\left[X_2>\sigma_2'\right]\middle|\left[X_1>\sigma_1'\right]\right\}=\Pr\left\{X_2>\sigma_2'\right\} \tag{4-22}$$

Substitution of Equations (4-21) and (4-22) into Equations (4-20a) and (4-20b) results in a single logical expression for the composite reliability:

$$R_c(\sigma_1,\sigma_2) = \Pr\{X_1 > \sigma_1'\}\Pr\{X_2 > \sigma_2'\} \tag{4-23}$$

The two Boolean terms in Equation (4-23) represent uniaxial reliabilities and, hence, the composite reliability in functional form becomes

$$R_c(\sigma_1,\sigma_2) = R_{X_1}(\sigma_1')R_{X_2}(\sigma_2') \tag{4-24}$$

The reliability domain and the joint failure pdf have not actually changed. However, the transformation of the stress space has recast their mathematical expressions in terms of the transformed variables.

In order to determine the joint failure pdf, the composite reliability is converted into the joint CDF for the composite, $F_c(\sigma_1,\sigma_2)$, since $F_c(\sigma_1,\sigma_2)$ is defined as the area integral of the joint failure pdf over the domains of the failure modes in the stress space. Since $F = 1-R$, Equation (4-24), expressed in terms of CDF's becomes

$$1-F_c(\sigma_1,\sigma_2) = \{1-F_{X_1}(\sigma_1')\}\{1-F_{X_2}(\sigma_2')\}$$

or,

$$F_c(\sigma_1,\sigma_2) = F_{X_1}(\sigma_1') + F_{X_2}(\sigma_2') - F_{X_1}(\sigma_1')F_{X_2}(\sigma_2') \tag{4-25}$$

Since $R_c(\sigma_1,\sigma_2)$ is invariant for both the physical and transformed stress spaces, $F_c(\sigma_1,\sigma_2)$ is also invariant for the two spaces.

The joint failure pdf in the transformed stress space, $f_{X_1,X_2}(\sigma_1',\sigma_2')$, may be obtained by differentiating the joint CDF by each of the transformed variables, σ_1' and σ_2':

$$f_{X_1,X_2}(\sigma_1',\sigma_2') = \frac{\partial^2}{\partial\sigma_1'\partial\sigma_2'}\left|F_c\right| \tag{4-26}$$

$$= \frac{\partial^2}{\partial\sigma_1'\partial\sigma_2'}\left[F_{X_1}(\sigma_1')F_{X_2}(\sigma_2') - F_{X_1}(\sigma_1') - F_{X_2}(\sigma_2')\right]$$

$$= \frac{\partial}{\partial\sigma_1'}\left[F_{X_1}(\sigma_1')f_{X_2}(\sigma_2') - f_{X_2}(\sigma_2')\right]$$

$$= f_{X_1}(\sigma_1')f_{X_2}(\sigma_2') \tag{4-27}$$

The absolute value is included in Equation (4-26) since the joint failure pdf must be a positive-valued function; the differentiation of reliability and failure probability result in positive-valued and negative-valued "mirror images" of the joint failure pdf because $F_c = 1 - R_c$.

The joint failure pdf in the physical stress space, $f_{(\mathcal{F}_1)_1,(\mathcal{F}_2)_2}(\sigma_1,\sigma_2)$, may similarly be obtained by differentiating the joint CDF by each of the physical variables, σ_1 and σ_2:

$$f_{(\mathcal{F}_1)_1,(\mathcal{F}_2)_2}(\sigma_1,\sigma_2) = \frac{\partial^2}{\partial\sigma_1\partial\sigma_2}\left|F_c\right|$$

$$
=\frac{\partial^2}{\partial\sigma_1\partial\sigma_2}\left[F_{X_1}(\sigma_1')F_{X_2}(\sigma_2')-F_{X_1}(\sigma_1')-F_{X_2}(\sigma_2')\right]
$$

$$
=\left(\frac{\partial\sigma_1'}{\partial\sigma_1}\right)\left(\frac{\partial\sigma_2'}{\partial\sigma_2}\right)f_{X_1}(\sigma_1')f_{X_2}(\sigma_2')+\left(\frac{\partial\sigma_1'}{\partial\sigma_2}\right)\left(\frac{\partial\sigma_2'}{\partial\sigma_1}\right)f_{X_1}(\sigma_1')f_{X_2}(\sigma_2')
$$

$$
+\left(\frac{\partial\sigma_1'}{\partial\sigma_2}\right)\left(\frac{\partial\sigma_1'}{\partial\sigma_1}\right)\frac{\partial f_{X_1}(\sigma_1')}{\partial\sigma_1}F_{X_2}(\sigma_2')+\left(\frac{\partial\sigma_2'}{\partial\sigma_2}\right)\left(\frac{\partial\sigma_2'}{\partial\sigma_1}\right)F_{X_1}(\sigma_1')\frac{\partial f_{X_2}(\sigma_2')}{\partial\sigma_2}
$$

$$
+\left(\frac{\partial^2\sigma_1'}{\partial\sigma_1\partial\sigma_2}\right)f_{X_1}(\sigma_1')F_{X_2}(\sigma_2')+\left(\frac{\partial^2\sigma_2'}{\partial\sigma_1\partial\sigma_2}\right)F_{X_1}(\sigma_1')f_{X_2}(\sigma_2')
$$

$$
-\left(\frac{\partial\sigma_1'}{\partial\sigma_1}\right)\left(\frac{\partial\sigma_1'}{\partial\sigma_2}\right)\frac{\partial f_{X_1}(\sigma_1')}{\partial\sigma_1}-\left(\frac{\partial\sigma_2'}{\partial\sigma_1}\right)\left(\frac{\partial\sigma_2'}{\partial\sigma_2}\right)\frac{\partial f_{X_2}(\sigma_2')}{\partial\sigma_2}
$$

$$
-\left(\frac{\partial^2\sigma_1'}{\partial\sigma_1\partial\sigma_2}\right)f_{X_1}(\sigma_1')-\left(\frac{\partial^2\sigma_2'}{\partial\sigma_1\partial\sigma_2}\right)f_{X_2}(\sigma_2') \tag{4-28}
$$

Substituting the relations of Equations (4-19a) through (4-19d) into Equation (4-28) gives an expression for the joint failure pdf in the physical stress space in terms of the uniaxial failure pdf's and the coupling functions:

$$
f_{(\mathcal{F}_1)_1,(\mathcal{F}_2)_2}(\sigma_1,\sigma_2)=(1)(1)f_{X_1}(\sigma_1')f_{X_2}(\sigma_2')+\left(-\frac{dC_{12}}{d\sigma_2}\right)\left(-\frac{dC_{21}}{d\sigma_1}\right)f_{X_1}(\sigma_1')f_{X_2}(\sigma_2')
$$

$$+\left(-\frac{dC_{12}}{d\sigma_2}\right)(1)\frac{\partial f_{X_1}(\sigma_1')}{\partial\sigma_1}F_{X_2}(\sigma_2')+(1)\left(-\frac{dC_{21}}{d\sigma_1}\right)F_{X_1}(\sigma_1')\frac{\partial f_{X_2}(\sigma_2')}{\partial\sigma_2}$$

$$+(0)f_{X_1}(\sigma_1')F_{X_2}(\sigma_2')+(0)F_{X_1}(\sigma_1')f_{X_2}(\sigma_2')$$

$$-(1)\left(-\frac{dC_{12}}{d\sigma_2}\right)\frac{\partial f_{X_1}(\sigma_1')}{\partial\sigma_1}-\left(-\frac{dC_{21}}{d\sigma_1}\right)(1)\frac{\partial f_{X_2}(\sigma_2')}{\partial\sigma_2}$$

$$-(0)f_{X_1}(\sigma_1')-(0)f_{X_2}(\sigma_2')$$

$$=f_{X_1}(\sigma_1')f_{X_2}(\sigma_2')+\left(\frac{dC_{12}}{d\sigma_2}\right)\left(\frac{dC_{21}}{d\sigma_1}\right)f_{X_1}(\sigma_1')f_{X_2}(\sigma_2')$$

$$-\left(\frac{dC_{12}}{d\sigma_2}\right)\frac{\partial f_{X_1}(\sigma_1')}{\partial\sigma_1}F_{X_2}(\sigma_2')-\left(\frac{dC_{21}}{d\sigma_1}\right)F_{X_1}(\sigma_1')\frac{\partial f_{X_2}(\sigma_2')}{\partial\sigma_2}$$

$$+\left(\frac{dC_{12}}{d\sigma_2}\right)\frac{\partial f_{X_1}(\sigma_1')}{\partial\sigma_1}+\left(\frac{dC_{21}}{d\sigma_1}\right)\frac{\partial f_{X_2}(\sigma_2')}{\partial\sigma_2} \tag{4-29}$$

In order to determine the interdependency of the physical random variables , $(\mathcal{F}_1)_1$ and $(\mathcal{F}_2)_2$, the conditional probabilities , $f_{(\mathcal{F}_1)_1|(\mathcal{F}_2)_2}(\sigma_1|\sigma_2)$ and $f_{(\mathcal{F}_2)_2|(\mathcal{F}_1)_1}(\sigma_2|\sigma_1)$, must be determined. The conditional probabilities may be found using the relations

$$f_{(\mathcal{F}_1)_1|(\mathcal{F}_2)_2}(\sigma_1|\sigma_2)=\frac{f_{(\mathcal{F}_1)_1,(\mathcal{F}_2)_2}(\sigma_1,\sigma_2)}{f_{X_2}(\sigma_2)} \tag{4-30}$$

and,

$$f_{(\mathcal{F}_2)_2|(\mathcal{F}_1)_1}(\sigma_2|\sigma_1)=\frac{f_{(\mathcal{F}_1)_1,(\mathcal{F}_2)_2}(\sigma_1,\sigma_2)}{f_{X_1}(\sigma_1)} \tag{4-31}$$

$(\mathcal{F}_1)_1$ and $(\mathcal{F}_2)_2$, are independent only if their conditional probabilities equal the equivalent uniaxial pdf's or,

$$f_{(\mathcal{F}_1)_1|(\mathcal{F}_2)_2}(\sigma_1|\sigma_2)=f_{X_1}(\sigma_1) \tag{4-32}$$

and,

$$f_{(\mathcal{F}_2)_2|(\mathcal{F}_1)_1}(\sigma_2|\sigma_1)=f_{X_2}(\sigma_2) \tag{4-33}$$

By substituting Equation (4-30) into Equation (4-32) or Equation (4-31) into Equation (4-32), a single independence criterion for $(\mathcal{F}_1)_1$ and $(\mathcal{F}_2)_2$ may be obtained:

$$f_{(\mathcal{F}_1)_1,(\mathcal{F}_2)_2}(\sigma_1,\sigma_2)=f_{X_1}(\sigma_1)f_{X_2}(\sigma_2) \tag{4-34}$$

$(\mathcal{F}_1)_1$ and $(\mathcal{F}_2)_2$ are only independent if there is no mechanistic coupling. The expression for the joint failure pdf in the physical stress space, Equation (4-29)

will reduce to the independence criterion for $(\mathcal{F}_1)_1$ and $(\mathcal{F}_2)_2$, Equation (4-34), only *if*

$$\frac{dC_{12}}{d\sigma_2} = \frac{dC_{21}}{d\sigma_1} = 0 \tag{4-35}$$

and, $\sigma_1{}' = \sigma_1$ and $\sigma_2{}' = \sigma_2$. These conditions are satisfied only when there is no mechanistic coupling, or $C_{12}(\sigma_2) = C_{21}(\sigma_1) = 0$. If mechanistic coupling is present, i.e., either $C_{12}(\sigma_2)$ or $C_{21}(\sigma_1)$ are nonzero, then Equation (4-29) will not reduce to Equation (4-34), and $(\mathcal{F}_1)_1$ and $(\mathcal{F}_2)_2$ cannot be independent random variables.

Equation (4-34) demonstrates that the conditions for independence of the intrinsic strength vectors in the biaxial stress space are only satisfied if there is no mechanistic coupling. To illustrate the explicit mathematical operations, this proof is repeated for explicitly defined linear coupling functions in Appendix B.

Both the joint CDF (joint probability of failure) and the reliability for the composite are determined by integrating the joint failure pdf over one or more specific domains in the stress space. The result of Equation (4-29) is evidence that when mechanistic coupling is present, the expression for the joint failure pdf is complicated and the evaluation of its area integral may require significant time and effort. Both the reliability and the joint CDF are the same in the physical and transformed stress spaces. Therefore, Equation (4-24) may be used in place of the joint failure pdf to calculate the reliability or joint CDF of a composite under combined stress provided the coupling functions and uniaxial CDF's are known. If the uniaxial CDF's, $F_{X_1}(\sigma_1)$ and $F_{X_2}(\sigma_2)$, are represented by Weibull distributions

(Equation (3-1)), then ,using Equation (4-24) and the relation R=1-F, the composite reliability may be expressed as

$$R_c(\sigma_1,\sigma_2)=\exp\left\{-\left[\frac{\sigma_1-C_{12}(\sigma_2)}{\beta_1}\right]^{\alpha_1}-\left[\frac{\sigma_2-C_{21}(\sigma_1)}{\beta_2}\right]^{\alpha_2}\right\} \tag{4-36}$$

and the joint CDF , or cumulative probability of failure, for the composite may be expressed as

$$F_c(\sigma_1,\sigma_2)=1-\exp\left\{-\left[\frac{\sigma_1-C_{12}(\sigma_2)}{\beta_1}\right]^{\alpha_1}-\left[\frac{\sigma_2-C_{21}(\sigma_1)}{\beta_2}\right]^{\alpha_2}\right\} \tag{4-37}$$

Equations (4-36) or (4-37) thus represent the probabilistic failure criterion in terms of Weibull distributions for biaxial combined tensile stress with mechanistically deterministic and probabilistically independent coupling of failure modes. Upon the identification of the mechanistic coupling function, either of these criteria may be used to evaluate the probability of failure of a structural element within which the stresses σ_1 and σ_2 are uniform.

2. Mechanistic Coupling Functions in Biaxial Tensile Combined Stress

For a composite under biaxial tensile combined stress, a mechanistic coupling function associated with each failure mode is needed. The analytical form for such a coupling function may be inferred from the micromechanical behavior of the composite under biaxial combined tensile stress. The effects of transverse

tension on the longitudinal strength are discussed first followed by a discussion of the effect of longitudinal tension on the transverse strength.

a. Longitudinal Strength Change Due to Transverse Stress

The effects of transverse loading on longitudinal strength for a composite specimen under biaxial tensile combined stress are illustrated in Figure 4-11. As the transverse component of the applied stress, σ_2, is increased, localized stress concentrations about microflaws within the matrix increase. These localized stress concentrations interfere with the transmission of the applied longitudinal loading from broken fibers to the adjacent fibers, effectively increasing the ineffective length, δ, of the composite. As δ increases, the number of adjacent fiber breaks within δ also increases and longitudinal failure will occur at a lower level of applied longitudinal loading.

When the transverse stress, σ_2=0, loading is uniaxial in the longitudinal direction and the longitudinal strength vector component, $(\mathcal{F}_1)_1$ is equal to the uniaxial longitudinal strength, X_1. In the limiting case, the transverse component of the applied loading is increased to a level such that the matrix is no longer capable of transmitting the longitudinal component of the loading to adjacent fibers, but not high enough to cause failure due to the matrix alone. In this case, δ becomes large and $(\mathcal{F}_1)_1$ approaches a minimum value. This minimum value is the fiber bundle strength, σ_b, which is the tensile strength of the composite fibers alone with no matrix to allow for local load sharing.

Since the transition from uniaxial longitudinal tensile stress to the bundle strength may be considered a gradual degradation of matrix properties, the usual partial fraction form of mixture equation employed in micromechanics is applicable. Such a partial fraction form can also be expressed in exponential form

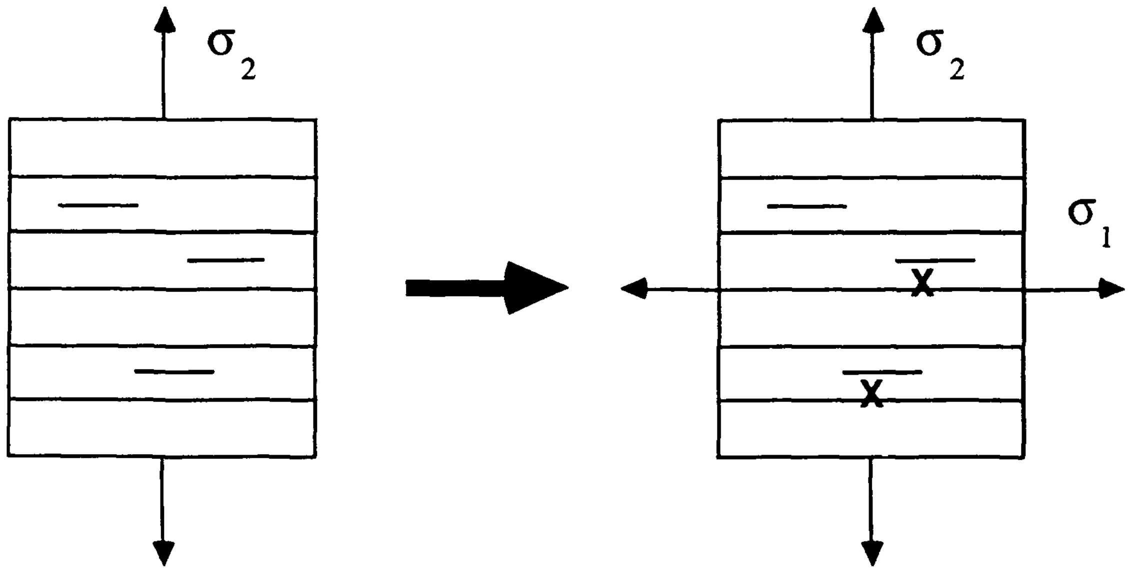


Figure 4-11. Micromechanical effect of transverse loading on longitudinal strength.

as derived in Appendix C. The exponential form is more convenient for operations in probability calculations and is adopted herein:

$$(\mathcal{F}_1)_1(\sigma_2) = X_1 - (X_1 - \sigma_b)\exp\left[-\left(\frac{1}{C_{12}\sigma_2}\right)\right] \tag{4-38}$$

where C_{12} is a constant coupling parameter. This model is shown graphically in Figure 4-12.

This model satisfies the limiting conditions. When $\sigma_2=0$, the exponential term is also equal to zero and therefore, $(\mathcal{F}_1)_1=X_1$. As σ_2 becomes large, the exponential term approaches unity, and $(\mathcal{F}_1)_1$ approaches the fiber bundle strength, σ_b. In addition, when the coupling parameter, C_{12}, is zero,

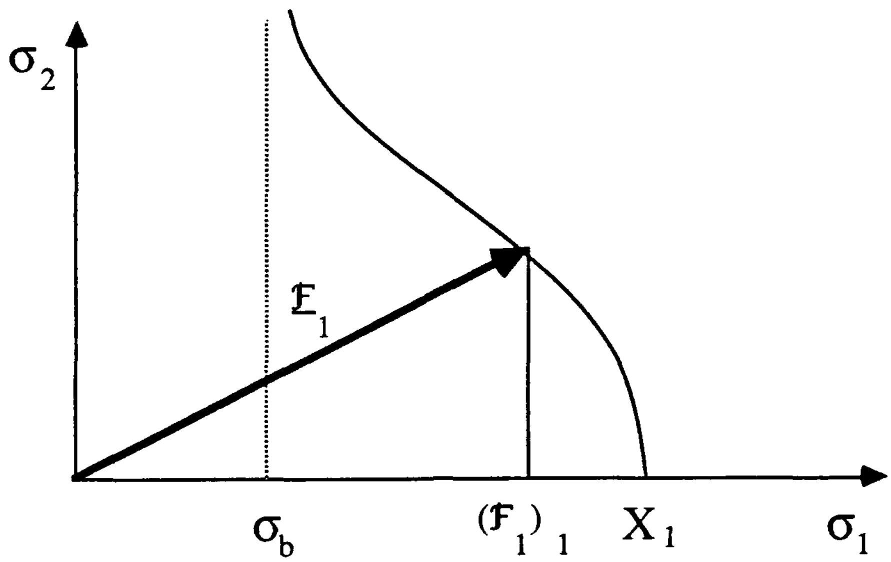


Figure 4-12. Effect of transverse stress on longitudinal strength in biaxial stress space.

longitudinal strength should be independent of transverse loading. If this condition is imposed on Equation (4-41), $(\mathcal{F}_1)_1 = X_1$, which is the required result.

b. Transverse Strength Change Due to Longitudinal Stress

The effect of longitudinal loading on transverse strength for biaxial tensile combined stress are illustrated in Figure 4-13. The matrix can be considered as containing a finite number of inherent microflaws which are randomly distributed throughout its volume. Uniaxial transverse loading does not create new microflaws, but instead increases the stress intensity about the pre-existing microflaws. Matrix failure will, therefore , emanate from from the largest pre-existing microflaw, since the stress intensity at that location will be highest. However, as the longitudinal component of the applied stress, σ_1, is increased, weak fibers begin to break, forming additional microflaws in the matrix

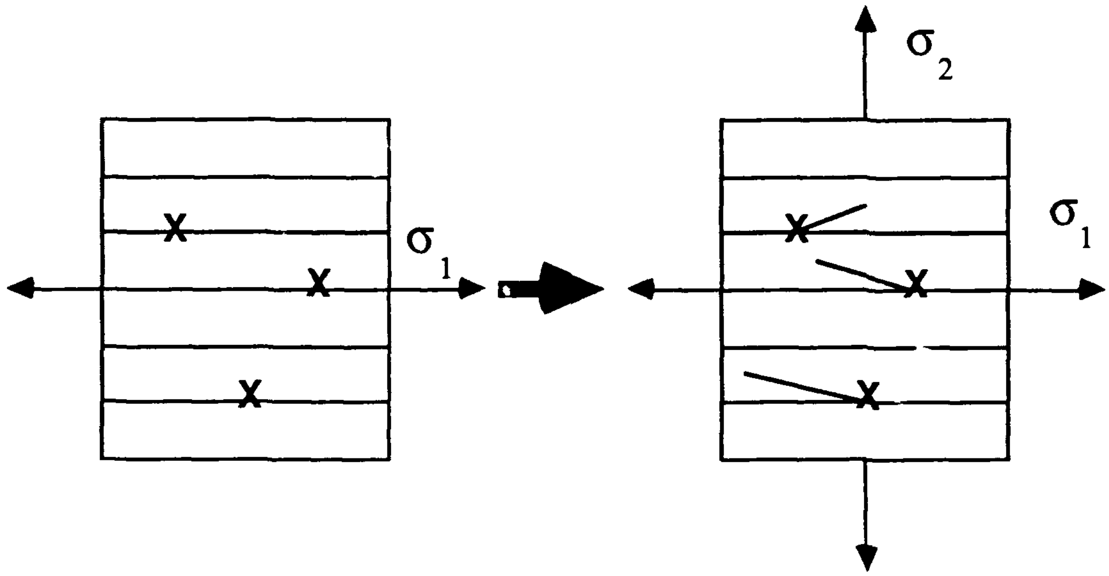


Figure 4-13. Micromechanical effect of longitudinal loading on transverse strength.

and further increasing the stress intensity in the vicinity of the fiber breaks, thereby introducing additional flaw sites and increase the probability of failure resulting in a lower transverse strength.

When the longitudinal stress, σ_1=0, loading is uniaxial in the transverse direction and the transverse strength vector component, $(\mathcal{F}_2)_2$ is equal to the uniaxial transverse strength, X_2. In the limiting case, the longitudinal component of the applied loading is increased to a level such that a large number of fiber breaks are present in the composite, but not high enough to cause failure due to fiber breaks alone. In this case, localized stress intensities become significantly high and $(\mathcal{F}_2)_2$ approaches zero.

A possible model for this variation in transverse strength is

$$(\mathcal{F}_2)_2(\sigma_1) = X_2\left[1-\exp\left(-\frac{1}{C_{21}\sigma_1}\right)\right] \tag{4-39}$$

where C_{21} is another coupling parameter. This model is displayed in Figure 4-14. This model satisfies the limiting conditions. When $\sigma_1=0$, the exponential term is also equal to zero and therefore, $(\mathcal{F}_2)_2=X_2$. As σ_1 becomes large, the exponential term approaches unity, and $(\mathcal{F}_2)_2$ approaches zero. Finally, when the coupling parameter, C_{21}, is zero, transverse strength should be independent of longitudinal loading. If this condition is imposed on Equation (4-39), $(\mathcal{F}_2)_2=X_2$, which is the expected result.

The combined stress probabilistic failure criterion for these coupling models may now be formulated. The composite reliability conditions for biaxial tensile stress have been given in Equation (4-10) as $(\mathcal{F}_1)_1>\sigma_1$ and $(\mathcal{F}_2)_2>\sigma_2$. When the results of Equations (4-38) and (4-39) are substituted for the intrinsic strength vector components and the reliability conditions are recast in terms of the uniaxial strengths, X_1 and X_2, the resulting inequalities,

$$X_1 > \frac{\sigma_1-\sigma_b\exp\left[-\left(\frac{1}{C_{12}\sigma_2}\right)\right]}{1-\exp\left[-\left(\frac{1}{C_{12}\sigma_2}\right)\right]} = g\left(\sigma_1,\sigma_2;\sigma_b,C_{12}\right) \tag{4-40}$$

and,

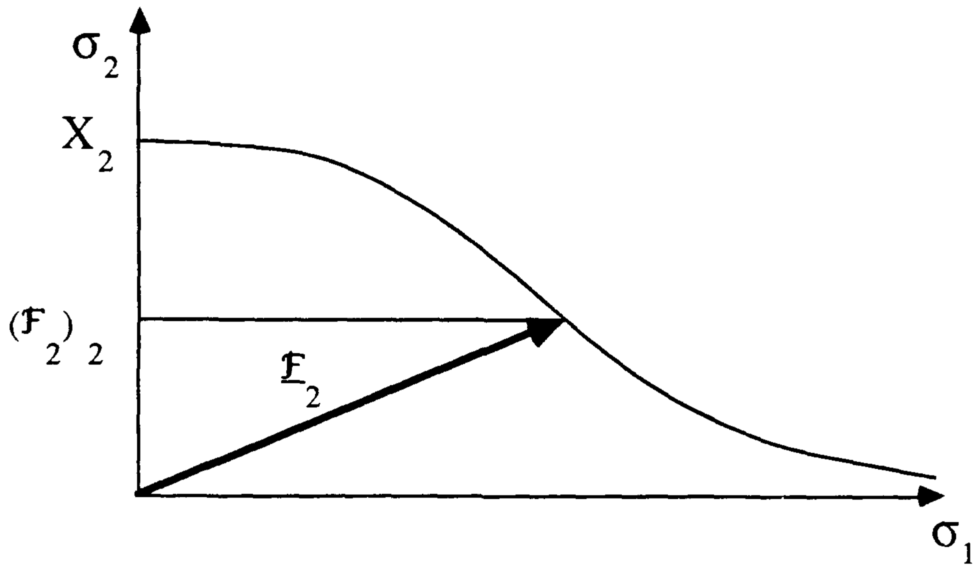


Figure 4-14. Effect of longitudinal stress on transverse strength in the biaxial stress space.

$$X_2 > \frac{\sigma_2}{1-\exp\left[-\left(\frac{1}{C_{21}\sigma_1}\right)\right]} = h(\sigma_1,\sigma_2;C_{21}) \tag{4-41}$$

provide the transformed stress components.

For uniaxial CDF's given by Weibull distributions, the probabilistic failure criterion in terms of reliability may then be expressed as

$$R_c(\sigma_1,\sigma_2)=\exp\left\{-\left[\frac{g(\sigma_1,\sigma_2;\sigma_b,C_{12})}{\beta_1}\right]^{\alpha_1}-\left[\frac{h(\sigma_1,\sigma_2;C_{21})}{\beta_2}\right]^{\alpha_2}\right\} \tag{4-42}$$

or, in terms of cumulative probability of failure, as

$$F_c(\sigma_1,\sigma_2)=1-\exp\left\{-\left[\frac{g(\sigma_1,\sigma_2;\sigma_b,C_{12})}{\beta_1}\right]^{\alpha_1}-\left[\frac{h(\sigma_1,\sigma_2;C_{21})}{\beta_2}\right]^{\alpha_2}\right\} \tag{4-43}$$

V. APPLICATION OF COMBINED STRESS FAILURE CRITERION

Reliability analysis or failure prediction for composite structures under multiaxial combined stress may be performed through the use of a post-processor for finite element stress analysis. Development of such a post-processor is a three-stage process. First, a probabilistic failure criterion for combined stress must be determined. Second, the structure must be subdivided into elements in order for the stress distribution within the structure to be determined using finite element stress analysis. Finally, the elemental stress data must be input to the probabilistic failure criterion and the resulting elemental reliabilities mathematically combined into an overall reliability or failure probability for the structure. This requires the definition of a load sharing model for the structure which defines the structural failure sequence.

The first stage in the development of a reliability post-processor is the determination of the combined stress probabilistic failure criterion for the composite as was performed in Chapter IV. The mechanistic coupling functions for each failure mode must first be defined. The analytical forms of these coupling functions are based on the micromechanical behavior of the composite under combined stress. Equations (4-41) and (4-42) are examples of mechanistic coupling functions for biaxial tensile combined stress. Each coupling function has one or more parameters which must be identified. Examples of possible parameters for coupling equations include the elastic moduli of the fiber and matrix, fiber bundle strength, matrix stress intensity factors, etc. Once the coupling functions are fully defined, Equation (4-27) may be used to formulate the

joint failure pdf for the composite. The joint failure pdf is then integrated over specific domains in the stress space associated with each failure mode to determine either the joint failure CDF or the composite reliability. Either of these functions constitute the probabilistic failure criterion for the composite and are coupled by the relation, $F_C=1-R_C$.

The second stage in the development is the establishment of an element grid for the composite structure so that the internal stress distribution may be determined via finite element analysis. The elements must be small enough that the stress distribution within the element is approximately homogeneous. The numerical simulations presented in Chapter III showed that the estimation of the intrinsic probabilistic strength of a composite structure based on the externally measured--or, in the case of finite element analysis, calculated--stress is erroneous when the stress distribution within the structure is heterogeneous. In addition, the size effects described in Chapter III must be taken into account in order for the reliability at a given location within the structure to be independent of the elemental grid. Corrections for the size effects may be incorporated into the parameters for the coupling equations.

When the stress data from the finite element element analysis are combined with the probabilistic failure criterion, structural reliabilities for each element are obtained. The overall reliability of the structure is a function of the elemental reliabilities. This function is defined by a load sharing model, which defines the failure sequence for the structure. An example of a load sharing model is the "weakest link" model, in which no load sharing between elements occurs and the entire structure fails when one element fails. In this model, the overall reliability of the structure is defined as the product of the elemental reliabilities. Thus, if a

rectangular grid of m elements by n elements is used, then the overall reliability for the structure for the 'weakest link' model is defined as

$$R=\prod_{i=1}^{mn} R_i \tag{5-1}$$

Composites are structurally redundant and all exhibit load sharing to some degree. While the "weakest link" model does not reflect the actual failure process in a composite, it does represent the weakest possible failure sequence for the structure and, hence, Equation (5-1) will result in a lower bound for the structural reliability of the composite.

VI. CONCLUSIONS AND RECOMMENDATIONS

This investigation motivated the need for an anisotropic probabilistic failure criterion for composites. This was performed by illustrating some of the physical and statistical phenomena which characterize probabilistic failure in composite structures through the use of numerical simulation. It was demonstrated that external load was not a sufficient measure of the internal strength when the internal stress distribution is nonuniform, and could only be used to characterize the internal strength in that case if the location of failure is known. In addition, some of the statistical size effects were illustrated for nonuniform stress distributions. It was also shown that a uniaxial probabilistic failure criterion is inadequate for characterizing failure in complex composite structures in which the internal stress distribution is multiaxial even when the applied loading is uniaxial. This was the motivation for formulating a probabilistic failure criterion for composites under combined stress conditions.

The essential elements of a combined stress probabilistic failure criterion were identified. These elements included the vectorial nature of applied stress and intrinsic strength, multiple failure modes with unique intrinsic strengths, the coupling of failure modes and the joint failure probability density function. The general formulation of a probabilistic failure criterion was performed by developing a logical representation of composite failure under combined stress using Boolean operations and converting this representation into functional form through the use of statistical distributions. It was demonstrated that the intrinsic strength vectors for a composite under biaxial tensile stress are independent only if the failure modes are uncoupled. Exponential coupling models for composites

under biaxial tensile combined stress were inferred from composite micromechanics. The essential elements of a finite element post-processor for multivariate structural reliability analysis was discussed.

Further investigations regarding the combined stress probabilistic failure criterion are recommended in two areas. First, the hypothesis that three of the four combinations of mechanistic and probabilistic coupling are externally indistinguishable should be tested through the use of numerical simulation. Second, the formulation of the combined stress probabilistic failure criterion should be expanded to include the effects of shear and compressive normal stresses.

APPENDIX A. SIMULATION SPREADSHEETS AND MACROS

Numerical simulation was performed using the Microsoft Excel™ spreadsheet with each elemental distribution having its own spreadsheet and controlling macro. Simulations for the "dogbone" specimens with one-dimensional elemental distributions were performed using the spreadsheet SIM9x1UNIAX and macro CONTROL9x1UNIAX. Simulations for the "dogbone" specimens with two-dimensional elemental distributions were performed using the spreadsheet SIM9x3UNIAX and macro CONTROL9x3UNIAX. Simulations for the plate model were performed using the spreadsheet SIM24x24UNIAX and macro CONTROL24x24UNILLS.

High strength variability was imposed by setting the Weibull shape parameter, α=5 and low strength variability was imposed by setting α=25. For the "dogbone" specimens, a heterogeneous stress distribution was imposed by using a curvature radius for the specimen of five and a homogeneous stress distribution was approximated by using a curvature radius of 100,000. In all simulations, the specimens were of a normalized thickness of 0.1 and the Weibull scale parameter, β, was held constant with a value of 100.

	A
1	Control9x1Uniax
2	=ERROR(FALSE)
3	=SELECT("R21C3")
4	=SET.VALUE(LocRslt9x1Uniax,REFTEXT(ACTIVE.CELL(),TRUE))
5	=SELECT("R21C4")
6	=SET.VALUE(StrRslt9x1Uniax,REFTEXT(ACTIVE.CELL(),TRUE))
7	=SELECT("R21C6")
8	=SET.VALUE(LoadRslt9x1Uniax,REFTEXT(ACTIVE.CELL(),TRUE))
9	=SELECT("r11c10:r19c10")
10	=COPY()
11	=SELECT("r11c6:r19c6")
12	=PASTE.SPECIAL(3,1)
13	=CALCULATION(3,FALSE)
14	=SET.VALUE(Load9x1Uniax,1)
15	=FORMULA("=Control!Load9x1Uniax","r5c2")
16	=CALCULATE.NOW()
17	=IF(DEREF('Simulation:Sim9x1Uniax'!$H$20)>=1,GOTO(End9x1Uniax))
18	=SET.VALUE(Load9x1Uniax,Load9x1Uniax+0.02)
19	=FORMULA("=Control!Load9x1Uniax","r5c2")
20	=CALCULATE.NOW()
21	=GOTO(A17)
22	=SELECT("R11C8:R19C8")
23	=FORMULA.FIND("1",2,1,2)
24	=SELECT("RC[-7]")
25	=COPY()
26	=SELECT(TEXTREF(LocRslt9x1Uniax,TRUE))
27	=PASTE.SPECIAL(3,1)
28	=SELECT("R11C8:R19C8")
29	=FORMULA.FIND("1",2,1,2)
30	=SELECT("RC[-4]")
31	=COPY()
32	=SELECT(TEXTREF(StrRslt9x1Uniax,TRUE))
33	=PASTE.SPECIAL(3,1)
34	=SELECT("R5C2")
35	=COPY()
36	=SELECT(TEXTREF(LoadRslt9x1Uniax,TRUE))
37	=PASTE.SPECIAL(3,1)
38	=CANCEL.COPY()
39	=SET.VALUE(LocRslt9x1Uniax,REFTEXT(OFFSET(TEXTREF(LocRslt9x1Uniax,TRUE),1,0),TRUE))
40	=SET.VALUE(StrRslt9x1Uniax,REFTEXT(OFFSET(TEXTREF(StrRslt9x1Uniax,TRUE),1,0),TRUE))
41	=SET.VALUE(LoadRslt9x1Uniax,REFTEXT(OFFSET(TEXTREF(LoadRslt9x1Uniax,TRUE),1,0),TRUE)
42	=GOTO(A9)
43	=ALERT("Simulation complete",3)
44	=RETURN()

	A	B	C	D
1	**Sim9x1Uniax**			
2	**Specimen Geometry**			
3	Thickness	0.1		
4	Width	1		
5	Radius	100000		
6	Load	=CONTROL!Load9x1Uniax		
7	**Material Strength**			
8	Alpha	5		
9	Beta	100		
10	**Location**	**Width**	**Area**	**Stress**
11				
12	1	=B4+2*B5*(1-COS(ASIN(((B4/2)*4)/B5)))	=B12*B3	=B6/C12
13	2	=B4+2*B5*(1-COS(ASIN(((B4/2)*3)/B5)))	=B13*B3	=B6/C13
14	3	=B4+2*B5*(1-COS(ASIN(((B4/2)*2)/B5)))	=B14*B3	=B6/C14
15	4	=B4+2*B5*(1-COS(ASIN(((B4/2)*1)/B5)))	=B15*B3	=B6/C15
16	5	=B4	=B16*B3	=B6/C16
17	6	=B4+2*B5*(1-COS(ASIN(((B4/2)*1)/B5)))	=B17*B3	=B6/C17
18	7	=B4+2*B5*(1-COS(ASIN(((B4/2)*2)/B5)))	=B18*B3	=B6/C18
19	8	=B4+2*B5*(1-COS(ASIN(((B4/2)*3)/B5)))	=B19*B3	=B6/C19
20	9	=B4+2*B5*(1-COS(ASIN(((B4/2)*4)/B5)))	=B20*B3	=B6/C20
21				

	E	F	G	H	I
1					
2					
3					
4					
5					
6					
7					
8					
9					
10	**Fractional**	**Intrinsic**	**Fail=1**	**Stress/**	
11	**Rank**	**Strength**		**Strength**	
12	0.58185181	=EXP((LN(-LN(1-E12))+B8*LN(B9))/B8)	=IF(D12<F12,0,1)	=D12/F12	=RAND()
13	0.57799672	=EXP((LN(-LN(1-E13))+B8*LN(B9))/B8)	=IF(D13<F13,0,1)	=D13/F13	=RAND()
14	0.71719553	=EXP((LN(-LN(1-E14))+B8*LN(B9))/B8)	=IF(D14<F14,0,1)	=D14/F14	=RAND()
15	0.78865708	=EXP((LN(-LN(1-E15))+B8*LN(B9))/B8)	=IF(D15<F15,0,1)	=D15/F15	=RAND()
16	0.61259493	=EXP((LN(-LN(1-E16))+B8*LN(B9))/B8)	=IF(D16<F16,0,1)	=D16/F16	=RAND()
17	0.50621835	=EXP((LN(-LN(1-E17))+B8*LN(B9))/B8)	=IF(D17<F17,0,1)	=D17/F17	=RAND()
18	0.78176811	=EXP((LN(-LN(1-E18))+B8*LN(B9))/B8)	=IF(D18<F18,0,1)	=D18/F18	=RAND()
19	0.95597515	=EXP((LN(-LN(1-E19))+B8*LN(B9))/B8)	=IF(D19<F19,0,1)	=D19/F19	=RAND()
20	0.84329615	=EXP((LN(-LN(1-E20))+B8*LN(B9))/B8)	=IF(D20<F20,0,1)	=D20/F20	=RAND()
21			=SUM(G12:G20)		

	B
1	Control9x3Uniax
2	=SELECT("R1C50")
3	=SET.VALUE(Result9x3Uniax,REFTEXT(ACTIVE.CELL(),TRUE))
4	=SELECT("r12c46:r20c48")
5	=COPY()
6	=SELECT("r12c23:r20c25")
7	=PASTE.SPECIAL(3,1)
8	=CALCULATION(3,FALSE)
9	=SET.VALUE(Load9x3Uniax,2)
10	=FORMULA("=Control!Load9x3Uniax","r5c2")
11	=CALCULATE.NOW()
12	=IF(DEREF('Simulation:Sim9x3Uniax'!$AP$21)>=1,GOTO(End9x3Uniax))
13	=SET.VALUE(Load9x3Uniax,Load9x3Uniax+0.02)
14	=FORMULA("=Control!Load9x3Uniax","r5c2")
15	=CALCULATE.NOW()
16	=GOTO(B12)
17	=SELECT("R5C2")
18	=COPY()
19	=SELECT(TEXTREF(Result9x3Uniax,TRUE))
20	=PASTE.SPECIAL(3,1)
21	=CANCEL.COPY()
22	=SET.VALUE(Result9x3Uniax,REFTEXT(OFFSET(TEXTREF(Result9x3Uniax,TRUE),1,0),TRUE))
23	=GOTO(B4)
24	=ALERT("Simulation complete",3)
25	=RETURN()

	A	B	C	D	E	F
1	**Sim9x3Uniax**					
2	**Specimen Geometry**					
3	Thickness	0.1				
4	Width	1				
5	Radius	5				
6	Load	=CONTROL!Load9x3Uniax				
7	**Material Strength**					
8	Alpha	5				
9	Beta	100				
10						
11	**Location**	**Width**	**Area**	**Applied**	**Fractional**	**Fractional**
12	**(i)**			**Stress**	**Rank (i1)**	**Rank (i2)**
13	1	=B4+2*B5*(1-COS(ASIN(((B4/2)*4)/B5)))	=B13*B3	=B6/C13	0.51859268	0.31009085
14	2	=B4+2*B5*(1-COS(ASIN(((B4/2)*3)/B5)))	=B14*B3	=B6/C14	0.93096839	0.25189417
15	3	=B4+2*B5*(1-COS(ASIN(((B4/2)*2)/B5)))	=B15*B3	=B6/C15	0.10456611	0.15516764
16	4	=B4+2*B5*(1-COS(ASIN(((B4/2)*1)/B5)))	=B16*B3	=B6/C16	0.28811085	0.74799963
17	5	=B4	=B17*B3	=B6/C17	0.38902830	0.85829602
18	6	=B4+2*B5*(1-COS(ASIN(((B4/2)*1)/B5)))	=B18*B3	=B6/C18	0.41510283	0.93626304
19	7	=B4+2*B5*(1-COS(ASIN(((B4/2)*2)/B5)))	=B19*B3	=B6/C19	0.52109865	0.92119873
20	8	=B4+2*B5*(1-COS(ASIN(((B4/2)*3)/B5)))	=B20*B3	=B6/C20	0.33982481	0.63083682
21	9	=B4+2*B5*(1-COS(ASIN(((B4/2)*4)/B5)))	=B21*B3	=B6/C21	0.15995920	0.17071646
22						

	G	H	I
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
11	Fractional	Strength	Strength
12	Rank (I3)	(I1)	(I2)
13	0.61365641	=EXP((LN(-LN(1-E13))+$B$8*LN($B$9))/$B$8)	=EXP((LN(-LN(1-F13))+$B$8*LN($B$9))/$B$8)
14	0.06403556	=EXP((LN(-LN(1-E14))+$B$8*LN($B$9))/$B$8)	=EXP((LN(-LN(1-F14))+$B$8*LN($B$9))/$B$8)
15	0.11272546	=EXP((LN(-LN(1-E15))+$B$8*LN($B$9))/$B$8)	=EXP((LN(-LN(1-F15))+$B$8*LN($B$9))/$B$8)
16	0.31577005	=EXP((LN(-LN(1-E16))+$B$8*LN($B$9))/$B$8)	=EXP((LN(-LN(1-F16))+$B$8*LN($B$9))/$B$8)
17	0.53662598	=EXP((LN(-LN(1-E17))+$B$8*LN($B$9))/$B$8)	=EXP((LN(-LN(1-F17))+$B$8*LN($B$9))/$B$8)
18	0.25071884	=EXP((LN(-LN(1-E18))+$B$8*LN($B$9))/$B$8)	=EXP((LN(-LN(1-F18))+$B$8*LN($B$9))/$B$8)
19	0.30405544	=EXP((LN(-LN(1-E19))+$B$8*LN($B$9))/$B$8)	=EXP((LN(-LN(1-F19))+$B$8*LN($B$9))/$B$8)
20	0.65980537	=EXP((LN(-LN(1-E20))+$B$8*LN($B$9))/$B$8)	=EXP((LN(-LN(1-F20))+$B$8*LN($B$9))/$B$8)
21	0.81776848	=EXP((LN(-LN(1-E21))+$B$8*LN($B$9))/$B$8)	=EXP((LN(-LN(1-F21))+$B$8*LN($B$9))/$B$8)
22			

	J	K
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
11	Strength	Fall
12	(I3)	7640
13	=EXP((LN(-LN(1-G13))+$B$8*LN($B$9))/$B$8)	=IF(AND(D13>J13,2*D13>I13,3*D13>H13),1,0)
14	=EXP((LN(-LN(1-G14))+$B$8*LN($B$9))/$B$8)	=IF(AND(D14>J14,2*D14>I14,3*D14>H14),1,0)
15	=EXP((LN(-LN(1-G15))+$B$8*LN($B$9))/$B$8)	=IF(AND(D15>J15,2*D15>I15,3*D15>H15),1,0)
16	=EXP((LN(-LN(1-G16))+$B$8*LN($B$9))/$B$8)	=IF(AND(D16>J16,2*D16>I16,3*D16>H16),1,0)
17	=EXP((LN(-LN(1-G17))+$B$8*LN($B$9))/$B$8)	=IF(AND(D17>J17,2*D17>I17,3*D17>H17),1,0)
18	=EXP((LN(-LN(1-G18))+$B$8*LN($B$9))/$B$8)	=IF(AND(D18>J18,2*D18>I18,3*D18>H18),1,0)
19	=EXP((LN(-LN(1-G19))+$B$8*LN($B$9))/$B$8)	=IF(AND(D19>J19,2*D19>I19,3*D19>H19),1,0)
20	=EXP((LN(-LN(1-G20))+$B$8*LN($B$9))/$B$8)	=IF(AND(D20>J20,2*D20>I20,3*D20>H20),1,0)
21	=EXP((LN(-LN(1-G21))+$B$8*LN($B$9))/$B$8)	=IF(AND(D21>J21,2*D21>I21,3*D21>H21),1,0)
22		

	L	M
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
11	Fail	Fail
12	7620	760
13	=IF(AND(D13>J13,D13>H13,3*D13>I13),1,0)	=IF(AND(D13>J13,D13>H13,2*D13>I13),1,0)
14	=IF(AND(D14>J14,D14>H14,3*D14>I14),1,0)	=IF(AND(D14>J14,D14>H14,2*D14>I14),1,0)
15	=IF(AND(D15>J15,D15>H15,3*D15>I15),1,0)	=IF(AND(D15>J15,D15>H15,2*D15>I15),1,0)
16	=IF(AND(D16>J16,D16>H16,3*D16>I16),1,0)	=IF(AND(D16>J16,D16>H16,2*D16>I16),1,0)
17	=IF(AND(D17>J17,D17>H17,3*D17>I17),1,0)	=IF(AND(D17>J17,D17>H17,2*D17>I17),1,0)
18	=IF(AND(D18>J18,D18>H18,3*D18>I18),1,0)	=IF(AND(D18>J18,D18>H18,2*D18>I18),1,0)
19	=IF(AND(D19>J19,D19>H19,3*D19>I19),1,0)	=IF(AND(D19>J19,D19>H19,2*D19>I19),1,0)
20	=IF(AND(D20>J20,D20>H20,3*D20>I20),1,0)	=IF(AND(D20>J20,D20>H20,2*D20>I20),1,0)
21	=IF(AND(D21>J21,D21>H21,3*D21>I21),1,0)	=IF(AND(D21>J21,D21>H21,2*D21>I21),1,0)
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11	Fail	Fail
12	7540	7510
13	=IF(AND(D13>I13,1.5*D13>J13,3*D13>H13),1,0)	=IF(AND(D13>I13,1.5*D13>H13,3*D13>J13),1,0)
14	=IF(AND(D14>I14,1.5*D14>J14,3*D14>H14),1,0)	=IF(AND(D14>I14,1.5*D14>H14,3*D14>J14),1,0)
15	=IF(AND(D15>I15,1.5*D15>J15,3*D15>H15),1,0)	=IF(AND(D15>I15,1.5*D15>H15,3*D15>J15),1,0)
16	=IF(AND(D16>I16,1.5*D16>J16,3*D16>H16),1,0)	=IF(AND(D16>I16,1.5*D16>H16,3*D16>J16),1,0)
17	=IF(AND(D17>I17,1.5*D17>J17,3*D17>H17),1,0)	=IF(AND(D17>I17,1.5*D17>H17,3*D17>J17),1,0)
18	=IF(AND(D18>I18,1.5*D18>J18,3*D18>H18),1,0)	=IF(AND(D18>I18,1.5*D18>H18,3*D18>J18),1,0)
19	=IF(AND(D19>I19,1.5*D19>J19,3*D19>H19),1,0)	=IF(AND(D19>I19,1.5*D19>H19,3*D19>J19),1,0)
20	=IF(AND(D20>I20,1.5*D20>J20,3*D20>H20),1,0)	=IF(AND(D20>I20,1.5*D20>H20,3*D20>J20),1,0)
21	=IF(AND(D21>I21,1.5*D21>J21,3*D21>H21),1,0)	=IF(AND(D21>I21,1.5*D21>H21,3*D21>J21),1,0)
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11	Fail	Fail
12	750	740
13	=IF(AND(D13>I13,1.5*D13>H13,1.5*D13>J13),1,0)	=IF(AND(D13>I13,D13>J13,3*D13>H13),1,0)
14	=IF(AND(D14>I14,1.5*D14>H14,1.5*D14>J14),1,0)	=IF(AND(D14>I14,D14>J14,3*D14>H14),1,0)
15	=IF(AND(D15>I15,1.5*D15>H15,1.5*D15>J15),1,0)	=IF(AND(D15>I15,D15>J15,3*D15>H15),1,0)
16	=IF(AND(D16>I16,1.5*D16>H16,1.5*D16>J16),1,0)	=IF(AND(D16>I16,D16>J16,3*D16>H16),1,0)
17	=IF(AND(D17>I17,1.5*D17>H17,1.5*D17>J17),1,0)	=IF(AND(D17>I17,D17>J17,3*D17>H17),1,0)
18	=IF(AND(D18>I18,1.5*D18>H18,1.5*D18>J18),1,0)	=IF(AND(D18>I18,D18>J18,3*D18>H18),1,0)
19	=IF(AND(D19>I19,1.5*D19>H19,1.5*D19>J19),1,0)	=IF(AND(D19>I19,D19>J19,3*D19>H19),1,0)
20	=IF(AND(D20>I20,1.5*D20>H20,1.5*D20>J20),1,0)	=IF(AND(D20>I20,D20>J20,3*D20>H20),1,0)
21	=IF(AND(D21>I21,1.5*D21>H21,1.5*D21>J21),1,0)	=IF(AND(D21>I21,D21>J21,3*D21>H21),1,0)
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11	**Fail**	**Fail**
12	**7320**	**7310**
13	=IF(AND(D13>H13,D13>J13,3*D13>I13),1,0)	=IF(AND(D13>H13,2*D13>I13,3*D13>J13),1,0)
14	=IF(AND(D14>H14,D14>J14,3*D14>I14),1,0)	=IF(AND(D14>H14,2*D14>I14,3*D14>J14),1,0)
15	=IF(AND(D15>H15,D15>J15,3*D15>I15),1,0)	=IF(AND(D15>H15,2*D15>I15,3*D15>J15),1,0)
16	=IF(AND(D16>H16,D16>J16,3*D16>I16),1,0)	=IF(AND(D16>H16,2*D16>I16,3*D16>J16),1,0)
17	=IF(AND(D17>H17,D17>J17,3*D17>I17),1,0)	=IF(AND(D17>H17,2*D17>I17,3*D17>J17),1,0)
18	=IF(AND(D18>H18,D18>J18,3*D18>I18),1,0)	=IF(AND(D18>H18,2*D18>I18,3*D18>J18),1,0)
19	=IF(AND(D19>H19,D19>J19,3*D19>I19),1,0)	=IF(AND(D19>H19,2*D19>I19,3*D19>J19),1,0)
20	=IF(AND(D20>H20,D20>J20,3*D20>I20),1,0)	=IF(AND(D20>H20,2*D20>I20,3*D20>J20),1,0)
21	=IF(AND(D21>H21,D21>J21,3*D21>I21),1,0)	=IF(AND(D21>H21,2*D21>I21,3*D21>J21),1,0)
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11	Fall	Fall
12	730	720
13	=IF(AND(D13>H13,D13>J13,2*D13>I13),1,0)	=IF(AND(D13>H13,D13>J13,3*D13>I13),1,0)
14	=IF(AND(D14>H14,D14>J14,2*D14>I14),1,0)	=IF(AND(D14>H14,D14>J14,3*D14>I14),1,0)
15	=IF(AND(D15>H15,D15>J15,2*D15>I15),1,0)	=IF(AND(D15>H15,D15>J15,3*D15>I15),1,0)
16	=IF(AND(D16>H16,D16>J16,2*D16>I16),1,0)	=IF(AND(D16>H16,D16>J16,3*D16>I16),1,0)
17	=IF(AND(D17>H17,D17>J17,2*D17>I17),1,0)	=IF(AND(D17>H17,D17>J17,3*D17>I17),1,0)
18	=IF(AND(D18>H18,D18>J18,2*D18>I18),1,0)	=IF(AND(D18>H18,D18>J18,3*D18>I18),1,0)
19	=IF(AND(D19>H19,D19>J19,2*D19>I19),1,0)	=IF(AND(D19>H19,D19>J19,3*D19>I19),1,0)
20	=IF(AND(D20>H20,D20>J20,2*D20>I20),1,0)	=IF(AND(D20>H20,D20>J20,3*D20>I20),1,0)
21	=IF(AND(D21>H21,D21>J21,2*D21>I21),1,0)	=IF(AND(D21>H21,D21>J21,3*D21>I21),1,0)
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11	**Fail**	**Fail**	**Failure**
12	**710**	**70**	**(Failure=1)**
13	=IF(AND(D13>H13,D13>I13,3*D13>J13),1,0)	=IF(AND(D13>H13,D13>I13,D13>J13),1,0)	=IF(SUM(K13:W13)<1,0,1)
14	=IF(AND(D14>H14,D14>I14,3*D14>J14),1,0)	=IF(AND(D14>H14,D14>I14,D14>J14),1,0)	=IF(SUM(K14:W14)<1,0,1)
15	=IF(AND(D15>H15,D15>I15,3*D15>J15),1,0)	=IF(AND(D15>H15,D15>I15,D15>J15),1,0)	=IF(SUM(K15:W15)<1,0,1)
16	=IF(AND(D16>H16,D16>I16,3*D16>J16),1,0)	=IF(AND(D16>H16,D16>I16,D16>J16),1,0)	=IF(SUM(K16:W16)<1,0,1)
17	=IF(AND(D17>H17,D17>I17,3*D17>J17),1,0)	=IF(AND(D17>H17,D17>I17,D17>J17),1,0)	=IF(SUM(K17:W17)<1,0,1)
18	=IF(AND(D18>H18,D18>I18,3*D18>J18),1,0)	=IF(AND(D18>H18,D18>I18,D18>J18),1,0)	=IF(SUM(K18:W18)<1,0,1)
19	=IF(AND(D19>H19,D19>I19,3*D19>J19),1,0)	=IF(AND(D19>H19,D19>I19,D19>J19),1,0)	=IF(SUM(K19:W19)<1,0,1)
20	=IF(AND(D20>H20,D20>I20,3*D20>J20),1,0)	=IF(AND(D20>H20,D20>I20,D20>J20),1,0)	=IF(SUM(K20:W20)<1,0,1)
21	=IF(AND(D21>H21,D21>I21,3*D21>J21),1,0)	=IF(AND(D21>H21,D21>I21,D21>J21),1,0)	=IF(SUM(K21:W21)<1,0,1)
22			=SUM(X13:X21)

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11	**Stress(I1)/**	**Stress(I2)/**	**Stress(I3)/**
12	**Strength(I1)**	**Strength(I2)**	**Strength(I3)**
13	=IF(D13/H13>1,"Failed",D13/H13)	=IF(D13/I13>1,"Failed",D13/I13)	=IF(D13/J13>1,"Failed",D13/J13)
14	=IF(D14/H14>1,"Failed",D14/H14)	=IF(D14/I14>1,"Failed",D14/I14)	=IF(D14/J14>1,"Failed",D14/J14)
15	=IF(D15/H15>1,"Failed",D15/H15)	=IF(D15/I15>1,"Failed",D15/I15)	=IF(D15/J15>1,"Failed",D15/J15)
16	=IF(D16/H16>1,"Failed",D16/H16)	=IF(D16/I16>1,"Failed",D16/I16)	=IF(D16/J16>1,"Failed",D16/J16)
17	=IF(D17/H17>1,"Failed",D17/H17)	=IF(D17/I17>1,"Failed",D17/I17)	=IF(D17/J17>1,"Failed",D17/J17)
18	=IF(D18/H18>1,"Failed",D18/H18)	=IF(D18/I18>1,"Failed",D18/I18)	=IF(D18/J18>1,"Failed",D18/J18)
19	=IF(D19/H19>1,"Failed",D19/H19)	=IF(D19/I19>1,"Failed",D19/I19)	=IF(D19/J19>1,"Failed",D19/J19)
20	=IF(D20/H20>1,"Failed",D20/H20)	=IF(D20/I20>1,"Failed",D20/I20)	=IF(D20/J20>1,"Failed",D20/J20)
21	=IF(D21/H21>1,"Failed",D21/H21)	=IF(D21/I21>1,"Failed",D21/I21)	=IF(D21/J21>1,"Failed",D21/J21)
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	D
1	Control24x24UniLLS
2	=ERROR(FALSE)
3	=CALCULATE.DOCUMENT()
4	=SELECT("R7C27")
5	=SET.VALUE(Result24x24UniLLS,REFTEXT(ACTIVE.CELL(),TRUE))
6	=SELECT("R32C52:R55C75")
7	=COPY()
8	=SELECT("R32C28:R55C51")
9	=PASTE.SPECIAL(3,1)
10	=CALCULATION(3,FALSE)
11	=SET.VALUE(Load24x24UniLLS,1.3)
12	=FORMULA(DEREF(D90),"R1C27")
13	=SELECT("R7C28:R30C51")
14	=FOR("COUNT",1,576)
15	=FORMULA("=EXP((LN(-LN(1-R[25]C))+R5C27*LN(R6C27))/R5C27)")
16	=SELECT(,"RC[1]")
17	=NEXT()
18	=SELECT("R57C2:R80C25")
19	=COPY()
20	=SELECT("R7C52:R30C75")
21	=PASTE()
22	=GOTO(D24)
23	=FORMULA.FILL(1)
24	=CALCULATE.DOCUMENT()
25	=SELECT("R7C1:R30C26")
26	=IF(NOT(FORMULA.FIND("1",2,1,1)),GOTO(Loop1_24x24UniLLS))
27	=SET.VALUE(Fail24x24UniLLS,REFTEXT(ACTIVE.CELL(),TRUE))
28	=SET.VALUE(Stress24x24UniLLS,DEREF(OFFSET(TEXTREF(Fail24x24UniLLS,TRUE),0,50)))
29	=SELECT(,"RC[-1]")
30	=WHILE(OR(ACTIVE.CELL()=1,ACTIVE.CELL()="F"))
31	=SELECT(,"RC[-1]")
32	=NEXT()
33	=IF(ACTIVE.CELL()=2,GOTO(Loop2_24x24UniLLS))
34	=DEREF(OFFSET(ACTIVE.CELL(),0,50))+0.5*Stress24x24UniLLS
35	=FORMULA(DEREF(D34),OFFSET(ACTIVE.CELL(),0,50))
36	=SELECT(,RELREF(TEXTREF(Fail24x24UniLLS,TRUE),ACTIVE.CELL()))
37	=FORMULA(0,OFFSET(ACTIVE.CELL(),0,26))
38	=FORMULA(0,OFFSET(ACTIVE.CELL(),0,50))
39	=SELECT(,"RC[1]")
40	=WHILE(OR(ACTIVE.CELL()=1,ACTIVE.CELL()="F"))
41	=SELECT(,"RC[1]")
42	=NEXT()
43	=IF(ACTIVE.CELL()=2,GOTO(Loop3_24x24UniLLS))
44	=DEREF(OFFSET(ACTIVE.CELL(),0,50))+0.5*Stress24x24UniLLS
45	=FORMULA(DEREF(D44),OFFSET(ACTIVE.CELL(),0,50))
46	=CALCULATE.DOCUMENT()
47	=GOTO(D25)
48	=SELECT(,RELREF(TEXTREF(Fail24x24UniLLS,TRUE),ACTIVE.CELL()))
49	=SELECT(,"RC[-1]")
50	=WHILE(ACTIVE.CELL()="F")
51	=SELECT(,"RC[-1]")
52	=NEXT()
53	=DEREF(OFFSET(ACTIVE.CELL(),0,50))+0.5*Stress24x24UniLLS
54	=FORMULA(DEREF(D53),OFFSET(ACTIVE.CELL(),0,50))
55	=CALCULATE.DOCUMENT()

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56	=GOTO(D25)
57	=SELECT(,RELREF(TEXTREF(Fail24x24UniLLS,TRUE),ACTIVE.CELL()))
58	=FORMULA(0,OFFSET(ACTIVE.CELL(),0,26))
59	=FORMULA(0,OFFSET(ACTIVE.CELL(),0,50))
60	=SELECT(,"RC[1]")
61	=WHILE(OR(ACTIVE.CELL()=1,ACTIVE.CELL()="F"))
62	=SELECT(,"RC[1]")
63	=NEXT()
64	=IF(ACTIVE.CELL()=2,GOTO(End24x24UniLLS))
65	=DEREF(OFFSET(ACTIVE.CELL(),0,50))+Stress24x24UniLLS
66	=FORMULA(DEREF(D65),OFFSET(ACTIVE.CELL(),0,50))
67	=CALCULATE.DOCUMENT()
68	=GOTO(D25)
69	=SET.VALUE(Load24x24UniLLS,Load24x24UniLLS+0.02)
70	=FORMULA(DEREF(D90),"R1C27")
71	=CALCULATE.DOCUMENT()
72	=GOTO(D25)
73	=SELECT(,RELREF(TEXTREF(Fail24x24UniLLS,TRUE),ACTIVE.CELL()))
74	=SELECT(,"RC[1]")
75	=WHILE(ACTIVE.CELL()<>2)
76	=FORMULA(0,OFFSET(ACTIVE.CELL(),0,26))
77	=FORMULA(0,OFFSET(ACTIVE.CELL(),0,50))
78	=SELECT(,"RC[1]")
79	=NEXT()
80	=CALCULATE.DOCUMENT()
81	=SELECT("R1C27")
82	=COPY()
83	=SELECT(TEXTREF(Result24x24UniLLS,TRUE))
84	=PASTE.SPECIAL(3,1)
85	=CANCEL.COPY()
86	=SET.VALUE(Result24x24UniLLS,REFTEXT(OFFSET(TEXTREF(Result24x24UniLLS,TRUE),1,0),TRU
87	=GOTO(D6)
88	=ALERT("Simulation complete",3)
89	=RETURN()

	A	B	C
1		Sim24x24Uniax-Plate Simulation	
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6		**Failure (SC) 0=OK 1,F=Fail 2=Boundary or Hole**	
7	2	=IF(ISERROR(B32/AB7),"F",(IF(B32<AB7,0,1)))	=IF(ISERROR(C32/AC7),"F",(IF(C32<AC7,0,1)))
8	2	=IF(ISERROR(B33/AB8),"F",(IF(B33<AB8,0,1)))	=IF(ISERROR(C33/AC8),"F",(IF(C33<AC8,0,1)))
9	2	=IF(ISERROR(B34/AB9),"F",(IF(B34<AB9,0,1)))	=IF(ISERROR(C34/AC9),"F",(IF(C34<AC9,0,1)))
10	2	=IF(ISERROR(B35/AB10),"F",(IF(B35<AB10,0,1)))	=IF(ISERROR(C35/AC10),"F",(IF(C35<AC10,0,1)))
11	2	=IF(ISERROR(B36/AB11),"F",(IF(B36<AB11,0,1)))	=IF(ISERROR(C36/AC11),"F",(IF(C36<AC11,0,1)))
12	2	=IF(ISERROR(B37/AB12),"F",(IF(B37<AB12,0,1)))	=IF(ISERROR(C37/AC12),"F",(IF(C37<AC12,0,1)))
13	2	=IF(ISERROR(B38/AB13),"F",(IF(B38<AB13,0,1)))	=IF(ISERROR(C38/AC13),"F",(IF(C38<AC13,0,1)))
14	2	=IF(ISERROR(B39/AB14),"F",(IF(B39<AB14,0,1)))	=IF(ISERROR(C39/AC14),"F",(IF(C39<AC14,0,1)))
15	2	=IF(ISERROR(B40/AB15),"F",(IF(B40<AB15,0,1)))	=IF(ISERROR(C40/AC15),"F",(IF(C40<AC15,0,1)))
16	2	=IF(ISERROR(B41/AB16),"F",(IF(B41<AB16,0,1)))	=IF(ISERROR(C41/AC16),"F",(IF(C41<AC16,0,1)))
17	2	=IF(ISERROR(B42/AB17),"F",(IF(B42<AB17,0,1)))	=IF(ISERROR(C42/AC17),"F",(IF(C42<AC17,0,1)))
18	2	=IF(ISERROR(B43/AB18),"F",(IF(B43<AB18,0,1)))	=IF(ISERROR(C43/AC18),"F",(IF(C43<AC18,0,1)))
19	2	=IF(ISERROR(B44/AB19),"F",(IF(B44<AB19,0,1)))	=IF(ISERROR(C44/AC19),"F",(IF(C44<AC19,0,1)))
20	2	=IF(ISERROR(B45/AB20),"F",(IF(B45<AB20,0,1)))	=IF(ISERROR(C45/AC20),"F",(IF(C45<AC20,0,1)))
21	2	=IF(ISERROR(B46/AB21),"F",(IF(B46<AB21,0,1)))	=IF(ISERROR(C46/AC21),"F",(IF(C46<AC21,0,1)))
22	2	=IF(ISERROR(B47/AB22),"F",(IF(B47<AB22,0,1)))	=IF(ISERROR(C47/AC22),"F",(IF(C47<AC22,0,1)))
23	2	=IF(ISERROR(B48/AB23),"F",(IF(B48<AB23,0,1)))	=IF(ISERROR(C48/AC23),"F",(IF(C48<AC23,0,1)))
24	2	=IF(ISERROR(B49/AB24),"F",(IF(B49<AB24,0,1)))	=IF(ISERROR(C49/AC24),"F",(IF(C49<AC24,0,1)))
25	2	=IF(ISERROR(B50/AB25),"F",(IF(B50<AB25,0,1)))	=IF(ISERROR(C50/AC25),"F",(IF(C50<AC25,0,1)))
26	2	=IF(ISERROR(B51/AB26),"F",(IF(B51<AB26,0,1)))	=IF(ISERROR(C51/AC26),"F",(IF(C51<AC26,0,1)))
27	2	=IF(ISERROR(B52/AB27),"F",(IF(B52<AB27,0,1)))	=IF(ISERROR(C52/AC27),"F",(IF(C52<AC27,0,1)))
28	2	=IF(ISERROR(B53/AB28),"F",(IF(B53<AB28,0,1)))	=IF(ISERROR(C53/AC28),"F",(IF(C53<AC28,0,1)))
29	2	=IF(ISERROR(B54/AB29),"F",(IF(B54<AB29,0,1)))	=IF(ISERROR(C54/AC29),"F",(IF(C54<AC29,0,1)))
30	2	=IF(ISERROR(B55/AB30),"F",(IF(B55<AB30,0,1)))	=IF(ISERROR(C55/AC30),"F",(IF(C55<AC30,0,1)))

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7	=IF(ISERROR(D32/AD7),"F",(IF(D32<AD7,0,1)))	=IF(ISERROR(E32/AE7),"F",(IF(E32<AE7,0,1)))
8	=IF(ISERROR(D33/AD8),"F",(IF(D33<AD8,0,1)))	=IF(ISERROR(E33/AE8),"F",(IF(E33<AE8,0,1)))
9	=IF(ISERROR(D34/AD9),"F",(IF(D34<AD9,0,1)))	=IF(ISERROR(E34/AE9),"F",(IF(E34<AE9,0,1)))
10	=IF(ISERROR(D35/AD10),"F",(IF(D35<AD10,0,1)))	=IF(ISERROR(E35/AE10),"F",(IF(E35<AE10,0,1)))
11	=IF(ISERROR(D36/AD11),"F",(IF(D36<AD11,0,1)))	=IF(ISERROR(E36/AE11),"F",(IF(E36<AE11,0,1)))
12	=IF(ISERROR(D37/AD12),"F",(IF(D37<AD12,0,1)))	=IF(ISERROR(E37/AE12),"F",(IF(E37<AE12,0,1)))
13	=IF(ISERROR(D38/AD13),"F",(IF(D38<AD13,0,1)))	=IF(ISERROR(E38/AE13),"F",(IF(E38<AE13,0,1)))
14	=IF(ISERROR(D39/AD14),"F",(IF(D39<AD14,0,1)))	=IF(ISERROR(E39/AE14),"F",(IF(E39<AE14,0,1)))
15	=IF(ISERROR(D40/AD15),"F",(IF(D40<AD15,0,1)))	=IF(ISERROR(E40/AE15),"F",(IF(E40<AE15,0,1)))
16	=IF(ISERROR(D41/AD16),"F",(IF(D41<AD16,0,1)))	=IF(ISERROR(E41/AE16),"F",(IF(E41<AE16,0,1)))
17	=IF(ISERROR(D42/AD17),"F",(IF(D42<AD17,0,1)))	=IF(ISERROR(E42/AE17),"F",(IF(E42<AE17,0,1)))
18	=IF(ISERROR(D43/AD18),"F",(IF(D43<AD18,0,1)))	=IF(ISERROR(E43/AE18),"F",(IF(E43<AE18,0,1)))
19	=IF(ISERROR(D44/AD19),"F",(IF(D44<AD19,0,1)))	=IF(ISERROR(E44/AE19),"F",(IF(E44<AE19,0,1)))
20	=IF(ISERROR(D45/AD20),"F",(IF(D45<AD20,0,1)))	=IF(ISERROR(E45/AE20),"F",(IF(E45<AE20,0,1)))
21	=IF(ISERROR(D46/AD21),"F",(IF(D46<AD21,0,1)))	=IF(ISERROR(E46/AE21),"F",(IF(E46<AE21,0,1)))
22	=IF(ISERROR(D47/AD22),"F",(IF(D47<AD22,0,1)))	=IF(ISERROR(E47/AE22),"F",(IF(E47<AE22,0,1)))
23	=IF(ISERROR(D48/AD23),"F",(IF(D48<AD23,0,1)))	=IF(ISERROR(E48/AE23),"F",(IF(E48<AE23,0,1)))
24	=IF(ISERROR(D49/AD24),"F",(IF(D49<AD24,0,1)))	=IF(ISERROR(E49/AE24),"F",(IF(E49<AE24,0,1)))
25	=IF(ISERROR(D50/AD25),"F",(IF(D50<AD25,0,1)))	=IF(ISERROR(E50/AE25),"F",(IF(E50<AE25,0,1)))
26	=IF(ISERROR(D51/AD26),"F",(IF(D51<AD26,0,1)))	=IF(ISERROR(E51/AE26),"F",(IF(E51<AE26,0,1)))
27	=IF(ISERROR(D52/AD27),"F",(IF(D52<AD27,0,1)))	=IF(ISERROR(E52/AE27),"F",(IF(E52<AE27,0,1)))
28	=IF(ISERROR(D53/AD28),"F",(IF(D53<AD28,0,1)))	=IF(ISERROR(E53/AE28),"F",(IF(E53<AE28,0,1)))
29	=IF(ISERROR(D54/AD29),"F",(IF(D54<AD29,0,1)))	=IF(ISERROR(E54/AE29),"F",(IF(E54<AE29,0,1)))
30	=IF(ISERROR(D55/AD30),"F",(IF(D55<AD30,0,1)))	=IF(ISERROR(E55/AE30),"F",(IF(E55<AE30,0,1)))

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7	=IF(ISERROR(F32/AF7),"F",(IF(F32<AF7,0,1)))	=IF(ISERROR(G32/AG7),"F",(IF(G32<AG7,0,1)))
8	=IF(ISERROR(F33/AF8),"F",(IF(F33<AF8,0,1)))	=IF(ISERROR(G33/AG8),"F",(IF(G33<AG8,0,1)))
9	=IF(ISERROR(F34/AF9),"F",(IF(F34<AF9,0,1)))	=IF(ISERROR(G34/AG9),"F",(IF(G34<AG9,0,1)))
10	=IF(ISERROR(F35/AF10),"F",(IF(F35<AF10,0,1)))	=IF(ISERROR(G35/AG10),"F",(IF(G35<AG10,0,1)))
11	=IF(ISERROR(F36/AF11),"F",(IF(F36<AF11,0,1)))	=IF(ISERROR(G36/AG11),"F",(IF(G36<AG11,0,1)))
12	=IF(ISERROR(F37/AF12),"F",(IF(F37<AF12,0,1)))	=IF(ISERROR(G37/AG12),"F",(IF(G37<AG12,0,1)))
13	=IF(ISERROR(F38/AF13),"F",(IF(F38<AF13,0,1)))	=IF(ISERROR(G38/AG13),"F",(IF(G38<AG13,0,1)))
14	=IF(ISERROR(F39/AF14),"F",(IF(F39<AF14,0,1)))	=IF(ISERROR(G39/AG14),"F",(IF(G39<AG14,0,1)))
15	=IF(ISERROR(F40/AF15),"F",(IF(F40<AF15,0,1)))	=IF(ISERROR(G40/AG15),"F",(IF(G40<AG15,0,1)))
16	=IF(ISERROR(F41/AF16),"F",(IF(F41<AF16,0,1)))	=IF(ISERROR(G41/AG16),"F",(IF(G41<AG16,0,1)))
17	=IF(ISERROR(F42/AF17),"F",(IF(F42<AF17,0,1)))	=IF(ISERROR(G42/AG17),"F",(IF(G42<AG17,0,1)))
18	=IF(ISERROR(F43/AF18),"F",(IF(F43<AF18,0,1)))	=IF(ISERROR(G43/AG18),"F",(IF(G43<AG18,0,1)))
19	=IF(ISERROR(F44/AF19),"F",(IF(F44<AF19,0,1)))	=IF(ISERROR(G44/AG19),"F",(IF(G44<AG19,0,1)))
20	=IF(ISERROR(F45/AF20),"F",(IF(F45<AF20,0,1)))	=IF(ISERROR(G45/AG20),"F",(IF(G45<AG20,0,1)))
21	=IF(ISERROR(F46/AF21),"F",(IF(F46<AF21,0,1)))	=IF(ISERROR(G46/AG21),"F",(IF(G46<AG21,0,1)))
22	=IF(ISERROR(F47/AF22),"F",(IF(F47<AF22,0,1)))	=IF(ISERROR(G47/AG22),"F",(IF(G47<AG22,0,1)))
23	=IF(ISERROR(F48/AF23),"F",(IF(F48<AF23,0,1)))	=IF(ISERROR(G48/AG23),"F",(IF(G48<AG23,0,1)))
24	=IF(ISERROR(F49/AF24),"F",(IF(F49<AF24,0,1)))	=IF(ISERROR(G49/AG24),"F",(IF(G49<AG24,0,1)))
25	=IF(ISERROR(F50/AF25),"F",(IF(F50<AF25,0,1)))	=IF(ISERROR(G50/AG25),"F",(IF(G50<AG25,0,1)))
26	=IF(ISERROR(F51/AF26),"F",(IF(F51<AF26,0,1)))	=IF(ISERROR(G51/AG26),"F",(IF(G51<AG26,0,1)))
27	=IF(ISERROR(F52/AF27),"F",(IF(F52<AF27,0,1)))	=IF(ISERROR(G52/AG27),"F",(IF(G52<AG27,0,1)))
28	=IF(ISERROR(F53/AF28),"F",(IF(F53<AF28,0,1)))	=IF(ISERROR(G53/AG28),"F",(IF(G53<AG28,0,1)))
29	=IF(ISERROR(F54/AF29),"F",(IF(F54<AF29,0,1)))	=IF(ISERROR(G54/AG29),"F",(IF(G54<AG29,0,1)))
30	=IF(ISERROR(F55/AF30),"F",(IF(F55<AF30,0,1)))	=IF(ISERROR(G55/AG30),"F",(IF(G55<AG30,0,1)))

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7	=IF(ISERROR(H32/AH7),"F",(IF(H32<AH7,0,1)))	=IF(ISERROR(I32/AI7),"F",(IF(I32<AI7,0,1)))
8	=IF(ISERROR(H33/AH8),"F",(IF(H33<AH8,0,1)))	=IF(ISERROR(I33/AI8),"F",(IF(I33<AI8,0,1)))
9	=IF(ISERROR(H34/AH9),"F",(IF(H34<AH9,0,1)))	=IF(ISERROR(I34/AI9),"F",(IF(I34<AI9,0,1)))
10	=IF(ISERROR(H35/AH10),"F",(IF(H35<AH10,0,1)))	=IF(ISERROR(I35/AI10),"F",(IF(I35<AI10,0,1)))
11	=IF(ISERROR(H36/AH11),"F",(IF(H36<AH11,0,1)))	=IF(ISERROR(I36/AI11),"F",(IF(I36<AI11,0,1)))
12	=IF(ISERROR(H37/AH12),"F",(IF(H37<AH12,0,1)))	=IF(ISERROR(I37/AI12),"F",(IF(I37<AI12,0,1)))
13	=IF(ISERROR(H38/AH13),"F",(IF(H38<AH13,0,1)))	=IF(ISERROR(I38/AI13),"F",(IF(I38<AI13,0,1)))
14	=IF(ISERROR(H39/AH14),"F",(IF(H39<AH14,0,1)))	=IF(ISERROR(I39/AI14),"F",(IF(I39<AI14,0,1)))
15	=IF(ISERROR(H40/AH15),"F",(IF(H40<AH15,0,1)))	=IF(ISERROR(I40/AI15),"F",(IF(I40<AI15,0,1)))
16	=IF(ISERROR(H41/AH16),"F",(IF(H41<AH16,0,1)))	=IF(ISERROR(I41/AI16),"F",(IF(I41<AI16,0,1)))
17	=IF(ISERROR(H42/AH17),"F",(IF(H42<AH17,0,1)))	2
18	=IF(ISERROR(H43/AH18),"F",(IF(H43<AH18,0,1)))	2
19	=IF(ISERROR(H44/AH19),"F",(IF(H44<AH19,0,1)))	2
20	=IF(ISERROR(H45/AH20),"F",(IF(H45<AH20,0,1)))	2
21	=IF(ISERROR(H46/AH21),"F",(IF(H46<AH21,0,1)))	=IF(ISERROR(I46/AI21),"F",(IF(I46<AI21,0,1)))
22	=IF(ISERROR(H47/AH22),"F",(IF(H47<AH22,0,1)))	=IF(ISERROR(I47/AI22),"F",(IF(I47<AI22,0,1)))
23	=IF(ISERROR(H48/AH23),"F",(IF(H48<AH23,0,1)))	=IF(ISERROR(I48/AI23),"F",(IF(I48<AI23,0,1)))
24	=IF(ISERROR(H49/AH24),"F",(IF(H49<AH24,0,1)))	=IF(ISERROR(I49/AI24),"F",(IF(I49<AI24,0,1)))
25	=IF(ISERROR(H50/AH25),"F",(IF(H50<AH25,0,1)))	=IF(ISERROR(I50/AI25),"F",(IF(I50<AI25,0,1)))
26	=IF(ISERROR(H51/AH26),"F",(IF(H51<AH26,0,1)))	=IF(ISERROR(I51/AI26),"F",(IF(I51<AI26,0,1)))
27	=IF(ISERROR(H52/AH27),"F",(IF(H52<AH27,0,1)))	=IF(ISERROR(I52/AI27),"F",(IF(I52<AI27,0,1)))
28	=IF(ISERROR(H53/AH28),"F",(IF(H53<AH28,0,1)))	=IF(ISERROR(I53/AI28),"F",(IF(I53<AI28,0,1)))
29	=IF(ISERROR(H54/AH29),"F",(IF(H54<AH29,0,1)))	=IF(ISERROR(I54/AI29),"F",(IF(I54<AI29,0,1)))
30	=IF(ISERROR(H55/AH30),"F",(IF(H55<AH30,0,1)))	=IF(ISERROR(I55/AI30),"F",(IF(I55<AI30,0,1)))

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7	=IF(ISERROR(J32/AJ7),"F",(IF(J32<AJ7,0,1)))	=IF(ISERROR(K32/AK7),"F",(IF(K32<AK7,0,1)))
8	=IF(ISERROR(J33/AJ8),"F",(IF(J33<AJ8,0,1)))	=IF(ISERROR(K33/AK8),"F",(IF(K33<AK8,0,1)))
9	=IF(ISERROR(J34/AJ9),"F",(IF(J34<AJ9,0,1)))	=IF(ISERROR(K34/AK9),"F",(IF(K34<AK9,0,1)))
10	=IF(ISERROR(J35/AJ10),"F",(IF(J35<AJ10,0,1)))	=IF(ISERROR(K35/AK10),"F",(IF(K35<AK10,0,1)))
11	=IF(ISERROR(J36/AJ11),"F",(IF(J36<AJ11,0,1)))	=IF(ISERROR(K36/AK11),"F",(IF(K36<AK11,0,1)))
12	=IF(ISERROR(J37/AJ12),"F",(IF(J37<AJ12,0,1)))	=IF(ISERROR(K37/AK12),"F",(IF(K37<AK12,0,1)))
13	=IF(ISERROR(J38/AJ13),"F",(IF(J38<AJ13,0,1)))	=IF(ISERROR(K38/AK13),"F",(IF(K38<AK13,0,1)))
14	=IF(ISERROR(J39/AJ14),"F",(IF(J39<AJ14,0,1)))	=IF(ISERROR(K39/AK14),"F",(IF(K39<AK14,0,1)))
15	=IF(ISERROR(J40/AJ15),"F",(IF(J40<AJ15,0,1)))	2
16	2	2
17	2	2
18	2	2
19	2	2
20	2	2
21	2	2
22	=IF(ISERROR(J47/AJ22),"F",(IF(J47<AJ22,0,1)))	2
23	=IF(ISERROR(J48/AJ23),"F",(IF(J48<AJ23,0,1)))	=IF(ISERROR(K48/AK23),"F",(IF(K48<AK23,0,1)))
24	=IF(ISERROR(J49/AJ24),"F",(IF(J49<AJ24,0,1)))	=IF(ISERROR(K49/AK24),"F",(IF(K49<AK24,0,1)))
25	=IF(ISERROR(J50/AJ25),"F",(IF(J50<AJ25,0,1)))	=IF(ISERROR(K50/AK25),"F",(IF(K50<AK25,0,1)))
26	=IF(ISERROR(J51/AJ26),"F",(IF(J51<AJ26,0,1)))	=IF(ISERROR(K51/AK26),"F",(IF(K51<AK26,0,1)))
27	=IF(ISERROR(J52/AJ27),"F",(IF(J52<AJ27,0,1)))	=IF(ISERROR(K52/AK27),"F",(IF(K52<AK27,0,1)))
28	=IF(ISERROR(J53/AJ28),"F",(IF(J53<AJ28,0,1)))	=IF(ISERROR(K53/AK28),"F",(IF(K53<AK28,0,1)))
29	=IF(ISERROR(J54/AJ29),"F",(IF(J54<AJ29,0,1)))	=IF(ISERROR(K54/AK29),"F",(IF(K54<AK29,0,1)))
30	=IF(ISERROR(J55/AJ30),"F",(IF(J55<AJ30,0,1)))	=IF(ISERROR(K55/AK30),"F",(IF(K55<AK30,0,1)))

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7	=IF(ISERROR(L32/AL7),"F",(IF(L32<AL7,0,1)))	=IF(ISERROR(M32/AM7),"F",(IF(M32<AM7,0,1)))
8	=IF(ISERROR(L33/AL8),"F",(IF(L33<AL8,0,1)))	=IF(ISERROR(M33/AM8),"F",(IF(M33<AM8,0,1)))
9	=IF(ISERROR(L34/AL9),"F",(IF(L34<AL9,0,1)))	=IF(ISERROR(M34/AM9),"F",(IF(M34<AM9,0,1)))
10	=IF(ISERROR(L35/AL10),"F",(IF(L35<AL10,0,1)))	=IF(ISERROR(M35/AM10),"F",(IF(M35<AM10,0,1)))
11	=IF(ISERROR(L36/AL11),"F",(IF(L36<AL11,0,1)))	=IF(ISERROR(M36/AM11),"F",(IF(M36<AM11,0,1)))
12	=IF(ISERROR(L37/AL12),"F",(IF(L37<AL12,0,1)))	=IF(ISERROR(M37/AM12),"F",(IF(M37<AM12,0,1)))
13	=IF(ISERROR(L38/AL13),"F",(IF(L38<AL13,0,1)))	=IF(ISERROR(M38/AM13),"F",(IF(M38<AM13,0,1)))
14	2	2
15	2	2
16	2	2
17	2	2
18	2	2
19	2	2
20	2	2
21	2	2
22	2	2
23	2	2
24	=IF(ISERROR(L49/AL24),"F",(IF(L49<AL24,0,1)))	=IF(ISERROR(M49/AM24),"F",(IF(M49<AM24,0,1)))
25	=IF(ISERROR(L50/AL25),"F",(IF(L50<AL25,0,1)))	=IF(ISERROR(M50/AM25),"F",(IF(M50<AM25,0,1)))
26	=IF(ISERROR(L51/AL26),"F",(IF(L51<AL26,0,1)))	=IF(ISERROR(M51/AM26),"F",(IF(M51<AM26,0,1)))
27	=IF(ISERROR(L52/AL27),"F",(IF(L52<AL27,0,1)))	=IF(ISERROR(M52/AM27),"F",(IF(M52<AM27,0,1)))
28	=IF(ISERROR(L53/AL28),"F",(IF(L53<AL28,0,1)))	=IF(ISERROR(M53/AM28),"F",(IF(M53<AM28,0,1)))
29	=IF(ISERROR(L54/AL29),"F",(IF(L54<AL29,0,1)))	=IF(ISERROR(M54/AM29),"F",(IF(M54<AM29,0,1)))
30	=IF(ISERROR(L55/AL30),"F",(IF(L55<AL30,0,1)))	=IF(ISERROR(M55/AM30),"F",(IF(M55<AM30,0,1)))

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7	=IF(ISERROR(N32/AN7),"F",(IF(N32<AN7,0,1)))	=IF(ISERROR(O32/AO7),"F",(IF(O32<AO7,0,1)))
8	=IF(ISERROR(N33/AN8),"F",(IF(N33<AN8,0,1)))	=IF(ISERROR(O33/AO8),"F",(IF(O33<AO8,0,1)))
9	=IF(ISERROR(N34/AN9),"F",(IF(N34<AN9,0,1)))	=IF(ISERROR(O34/AO9),"F",(IF(O34<AO9,0,1)))
10	=IF(ISERROR(N35/AN10),"F",(IF(N35<AN10,0,1)))	=IF(ISERROR(O35/AO10),"F",(IF(O35<AO10,0,1)))
11	=IF(ISERROR(N36/AN11),"F",(IF(N36<AN11,0,1)))	=IF(ISERROR(O36/AO11),"F",(IF(O36<AO11,0,1)))
12	=IF(ISERROR(N37/AN12),"F",(IF(N37<AN12,0,1)))	=IF(ISERROR(O37/AO12),"F",(IF(O37<AO12,0,1)))
13	=IF(ISERROR(N38/AN13),"F",(IF(N38<AN13,0,1)))	=IF(ISERROR(O38/AO13),"F",(IF(O38<AO13,0,1)))
14	2	2
15	2	2
16	2	2
17	2	2
18	2	2
19	2	2
20	2	2
21	2	2
22	2	2
23	2	2
24	=IF(ISERROR(N49/AN24),"F",(IF(N49<AN24,0,1)))	=IF(ISERROR(O49/AO24),"F",(IF(O49<AO24,0,1)))
25	=IF(ISERROR(N50/AN25),"F",(IF(N50<AN25,0,1)))	=IF(ISERROR(O50/AO25),"F",(IF(O50<AO25,0,1)))
26	=IF(ISERROR(N51/AN26),"F",(IF(N51<AN26,0,1)))	=IF(ISERROR(O51/AO26),"F",(IF(O51<AO26,0,1)))
27	=IF(ISERROR(N52/AN27),"F",(IF(N52<AN27,0,1)))	=IF(ISERROR(O52/AO27),"F",(IF(O52<AO27,0,1)))
28	=IF(ISERROR(N53/AN28),"F",(IF(N53<AN28,0,1)))	=IF(ISERROR(O53/AO28),"F",(IF(O53<AO28,0,1)))
29	=IF(ISERROR(N54/AN29),"F",(IF(N54<AN29,0,1)))	=IF(ISERROR(O54/AO29),"F",(IF(O54<AO29,0,1)))
30	=IF(ISERROR(N55/AN30),"F",(IF(N55<AN30,0,1)))	=IF(ISERROR(O55/AO30),"F",(IF(O55<AO30,0,1)))

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7	=IF(ISERROR(P32/AP7),"F",(IF(P32<AP7,0,1)))	=IF(ISERROR(Q32/AQ7),"F",(IF(Q32<AQ7,0,1)))
8	=IF(ISERROR(P33/AP8),"F",(IF(P33<AP8,0,1)))	=IF(ISERROR(Q33/AQ8),"F",(IF(Q33<AQ8,0,1)))
9	=IF(ISERROR(P34/AP9),"F",(IF(P34<AP9,0,1)))	=IF(ISERROR(Q34/AQ9),"F",(IF(Q34<AQ9,0,1)))
10	=IF(ISERROR(P35/AP10),"F",(IF(P35<AP10,0,1)))	=IF(ISERROR(Q35/AQ10),"F",(IF(Q35<AQ10,0,1)))
11	=IF(ISERROR(P36/AP11),"F",(IF(P36<AP11,0,1)))	=IF(ISERROR(Q36/AQ11),"F",(IF(Q36<AQ11,0,1)))
12	=IF(ISERROR(P37/AP12),"F",(IF(P37<AP12,0,1)))	=IF(ISERROR(Q37/AQ12),"F",(IF(Q37<AQ12,0,1)))
13	=IF(ISERROR(P38/AP13),"F",(IF(P38<AP13,0,1)))	=IF(ISERROR(Q38/AQ13),"F",(IF(Q38<AQ13,0,1)))
14	=IF(ISERROR(P39/AP14),"F",(IF(P39<AP14,0,1)))	=IF(ISERROR(Q39/AQ14),"F",(IF(Q39<AQ14,0,1)))
15	2	=IF(ISERROR(Q40/AQ15),"F",(IF(Q40<AQ15,0,1)))
16	2	2
17	2	2
18	2	2
19	2	2
20	2	2
21	2	2
22	2	=IF(ISERROR(Q47/AQ22),"F",(IF(Q47<AQ22,0,1)))
23	=IF(ISERROR(P48/AP23),"F",(IF(P48<AP23,0,1)))	=IF(ISERROR(Q48/AQ23),"F",(IF(Q48<AQ23,0,1)))
24	=IF(ISERROR(P49/AP24),"F",(IF(P49<AP24,0,1)))	=IF(ISERROR(Q49/AQ24),"F",(IF(Q49<AQ24,0,1)))
25	=IF(ISERROR(P50/AP25),"F",(IF(P50<AP25,0,1)))	=IF(ISERROR(Q50/AQ25),"F",(IF(Q50<AQ25,0,1)))
26	=IF(ISERROR(P51/AP26),"F",(IF(P51<AP26,0,1)))	=IF(ISERROR(Q51/AQ26),"F",(IF(Q51<AQ26,0,1)))
27	=IF(ISERROR(P52/AP27),"F",(IF(P52<AP27,0,1)))	=IF(ISERROR(Q52/AQ27),"F",(IF(Q52<AQ27,0,1)))
28	=IF(ISERROR(P53/AP28),"F",(IF(P53<AP28,0,1)))	=IF(ISERROR(Q53/AQ28),"F",(IF(Q53<AQ28,0,1)))
29	=IF(ISERROR(P54/AP29),"F",(IF(P54<AP29,0,1)))	=IF(ISERROR(Q54/AQ29),"F",(IF(Q54<AQ29,0,1)))
30	=IF(ISERROR(P55/AP30),"F",(IF(P55<AP30,0,1)))	=IF(ISERROR(Q55/AQ30),"F",(IF(Q55<AQ30,0,1)))

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7	=IF(ISERROR(R32/AR7),"F",(IF(R32<AR7,0,1)))	=IF(ISERROR(S32/AS7),"F",(IF(S32<AS7,0,1)))
8	=IF(ISERROR(R33/AR8),"F",(IF(R33<AR8,0,1)))	=IF(ISERROR(S33/AS8),"F",(IF(S33<AS8,0,1)))
9	=IF(ISERROR(R34/AR9),"F",(IF(R34<AR9,0,1)))	=IF(ISERROR(S34/AS9),"F",(IF(S34<AS9,0,1)))
10	=IF(ISERROR(R35/AR10),"F",(IF(R35<AR10,0,1)))	=IF(ISERROR(S35/AS10),"F",(IF(S35<AS10,0,1)))
11	=IF(ISERROR(R36/AR11),"F",(IF(R36<AR11,0,1)))	=IF(ISERROR(S36/AS11),"F",(IF(S36<AS11,0,1)))
12	=IF(ISERROR(R37/AR12),"F",(IF(R37<AR12,0,1)))	=IF(ISERROR(S37/AS12),"F",(IF(S37<AS12,0,1)))
13	=IF(ISERROR(R38/AR13),"F",(IF(R38<AR13,0,1)))	=IF(ISERROR(S38/AS13),"F",(IF(S38<AS13,0,1)))
14	=IF(ISERROR(R39/AR14),"F",(IF(R39<AR14,0,1)))	=IF(ISERROR(S39/AS14),"F",(IF(S39<AS14,0,1)))
15	=IF(ISERROR(R40/AR15),"F",(IF(R40<AR15,0,1)))	=IF(ISERROR(S40/AS15),"F",(IF(S40<AS15,0,1)))
16	=IF(ISERROR(R41/AR16),"F",(IF(R41<AR16,0,1)))	=IF(ISERROR(S41/AS16),"F",(IF(S41<AS16,0,1)))
17	2	=IF(ISERROR(S42/AS17),"F",(IF(S42<AS17,0,1)))
18	2	=IF(ISERROR(S43/AS18),"F",(IF(S43<AS18,0,1)))
19	2	=IF(ISERROR(S44/AS19),"F",(IF(S44<AS19,0,1)))
20	2	=IF(ISERROR(S45/AS20),"F",(IF(S45<AS20,0,1)))
21	=IF(ISERROR(R46/AR21),"F",(IF(R46<AR21,0,1)))	=IF(ISERROR(S46/AS21),"F",(IF(S46<AS21,0,1)))
22	=IF(ISERROR(R47/AR22),"F",(IF(R47<AR22,0,1)))	=IF(ISERROR(S47/AS22),"F",(IF(S47<AS22,0,1)))
23	=IF(ISERROR(R48/AR23),"F",(IF(R48<AR23,0,1)))	=IF(ISERROR(S48/AS23),"F",(IF(S48<AS23,0,1)))
24	=IF(ISERROR(R49/AR24),"F",(IF(R49<AR24,0,1)))	=IF(ISERROR(S49/AS24),"F",(IF(S49<AS24,0,1)))
25	=IF(ISERROR(R50/AR25),"F",(IF(R50<AR25,0,1)))	=IF(ISERROR(S50/AS25),"F",(IF(S50<AS25,0,1)))
26	=IF(ISERROR(R51/AR26),"F",(IF(R51<AR26,0,1)))	=IF(ISERROR(S51/AS26),"F",(IF(S51<AS26,0,1)))
27	=IF(ISERROR(R52/AR27),"F",(IF(R52<AR27,0,1)))	=IF(ISERROR(S52/AS27),"F",(IF(S52<AS27,0,1)))
28	=IF(ISERROR(R53/AR28),"F",(IF(R53<AR28,0,1)))	=IF(ISERROR(S53/AS28),"F",(IF(S53<AS28,0,1)))
29	=IF(ISERROR(R54/AR29),"F",(IF(R54<AR29,0,1)))	=IF(ISERROR(S54/AS29),"F",(IF(S54<AS29,0,1)))
30	=IF(ISERROR(R55/AR30),"F",(IF(R55<AR30,0,1)))	=IF(ISERROR(S55/AS30),"F",(IF(S55<AS30,0,1)))

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7	=IF(ISERROR(T32/AT7),"F",(IF(T32<AT7,0,1)))	=IF(ISERROR(U32/AU7),"F",(IF(U32<AU7,0,1)))
8	=IF(ISERROR(T33/AT8),"F",(IF(T33<AT8,0,1)))	=IF(ISERROR(U33/AU8),"F",(IF(U33<AU8,0,1)))
9	=IF(ISERROR(T34/AT9),"F",(IF(T34<AT9,0,1)))	=IF(ISERROR(U34/AU9),"F",(IF(U34<AU9,0,1)))
10	=IF(ISERROR(T35/AT10),"F",(IF(T35<AT10,0,1)))	=IF(ISERROR(U35/AU10),"F",(IF(U35<AU10,0,1)))
11	=IF(ISERROR(T36/AT11),"F",(IF(T36<AT11,0,1)))	=IF(ISERROR(U36/AU11),"F",(IF(U36<AU11,0,1)))
12	=IF(ISERROR(T37/AT12),"F",(IF(T37<AT12,0,1)))	=IF(ISERROR(U37/AU12),"F",(IF(U37<AU12,0,1)))
13	=IF(ISERROR(T38/AT13),"F",(IF(T38<AT13,0,1)))	=IF(ISERROR(U38/AU13),"F",(IF(U38<AU13,0,1)))
14	=IF(ISERROR(T39/AT14),"F",(IF(T39<AT14,0,1)))	=IF(ISERROR(U39/AU14),"F",(IF(U39<AU14,0,1)))
15	=IF(ISERROR(T40/AT15),"F",(IF(T40<AT15,0,1)))	=IF(ISERROR(U40/AU15),"F",(IF(U40<AU15,0,1)))
16	=IF(ISERROR(T41/AT16),"F",(IF(T41<AT16,0,1)))	=IF(ISERROR(U41/AU16),"F",(IF(U41<AU16,0,1)))
17	=IF(ISERROR(T42/AT17),"F",(IF(T42<AT17,0,1)))	=IF(ISERROR(U42/AU17),"F",(IF(U42<AU17,0,1)))
18	=IF(ISERROR(T43/AT18),"F",(IF(T43<AT18,0,1)))	=IF(ISERROR(U43/AU18),"F",(IF(U43<AU18,0,1)))
19	=IF(ISERROR(T44/AT19),"F",(IF(T44<AT19,0,1)))	=IF(ISERROR(U44/AU19),"F",(IF(U44<AU19,0,1)))
20	=IF(ISERROR(T45/AT20),"F",(IF(T45<AT20,0,1)))	=IF(ISERROR(U45/AU20),"F",(IF(U45<AU20,0,1)))
21	=IF(ISERROR(T46/AT21),"F",(IF(T46<AT21,0,1)))	=IF(ISERROR(U46/AU21),"F",(IF(U46<AU21,0,1)))
22	=IF(ISERROR(T47/AT22),"F",(IF(T47<AT22,0,1)))	=IF(ISERROR(U47/AU22),"F",(IF(U47<AU22,0,1)))
23	=IF(ISERROR(T48/AT23),"F",(IF(T48<AT23,0,1)))	=IF(ISERROR(U48/AU23),"F",(IF(U48<AU23,0,1)))
24	=IF(ISERROR(T49/AT24),"F",(IF(T49<AT24,0,1)))	=IF(ISERROR(U49/AU24),"F",(IF(U49<AU24,0,1)))
25	=IF(ISERROR(T50/AT25),"F",(IF(T50<AT25,0,1)))	=IF(ISERROR(U50/AU25),"F",(IF(U50<AU25,0,1)))
26	=IF(ISERROR(T51/AT26),"F",(IF(T51<AT26,0,1)))	=IF(ISERROR(U51/AU26),"F",(IF(U51<AU26,0,1)))
27	=IF(ISERROR(T52/AT27),"F",(IF(T52<AT27,0,1)))	=IF(ISERROR(U52/AU27),"F",(IF(U52<AU27,0,1)))
28	=IF(ISERROR(T53/AT28),"F",(IF(T53<AT28,0,1)))	=IF(ISERROR(U53/AU28),"F",(IF(U53<AU28,0,1)))
29	=IF(ISERROR(T54/AT29),"F",(IF(T54<AT29,0,1)))	=IF(ISERROR(U54/AU29),"F",(IF(U54<AU29,0,1)))
30	=IF(ISERROR(T55/AT30),"F",(IF(T55<AT30,0,1)))	=IF(ISERROR(U55/AU30),"F",(IF(U55<AU30,0,1)))

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7	=IF(ISERROR(V32/AV7),"F",(IF(V32<AV7,0,1)))	=IF(ISERROR(W32/AW7),"F",(IF(W32<AW7,0,1)))
8	=IF(ISERROR(V33/AV8),"F",(IF(V33<AV8,0,1)))	=IF(ISERROR(W33/AW8),"F",(IF(W33<AW8,0,1)))
9	=IF(ISERROR(V34/AV9),"F",(IF(V34<AV9,0,1)))	=IF(ISERROR(W34/AW9),"F",(IF(W34<AW9,0,1)))
10	=IF(ISERROR(V35/AV10),"F",(IF(V35<AV10,0,1)))	=IF(ISERROR(W35/AW10),"F",(IF(W35<AW10,0,1)))
11	=IF(ISERROR(V36/AV11),"F",(IF(V36<AV11,0,1)))	=IF(ISERROR(W36/AW11),"F",(IF(W36<AW11,0,1)))
12	=IF(ISERROR(V37/AV12),"F",(IF(V37<AV12,0,1)))	=IF(ISERROR(W37/AW12),"F",(IF(W37<AW12,0,1)))
13	=IF(ISERROR(V38/AV13),"F",(IF(V38<AV13,0,1)))	=IF(ISERROR(W38/AW13),"F",(IF(W38<AW13,0,1)))
14	=IF(ISERROR(V39/AV14),"F",(IF(V39<AV14,0,1)))	=IF(ISERROR(W39/AW14),"F",(IF(W39<AW14,0,1)))
15	=IF(ISERROR(V40/AV15),"F",(IF(V40<AV15,0,1)))	=IF(ISERROR(W40/AW15),"F",(IF(W40<AW15,0,1)))
16	=IF(ISERROR(V41/AV16),"F",(IF(V41<AV16,0,1)))	=IF(ISERROR(W41/AW16),"F",(IF(W41<AW16,0,1)))
17	=IF(ISERROR(V42/AV17),"F",(IF(V42<AV17,0,1)))	=IF(ISERROR(W42/AW17),"F",(IF(W42<AW17,0,1)))
18	=IF(ISERROR(V43/AV18),"F",(IF(V43<AV18,0,1)))	=IF(ISERROR(W43/AW18),"F",(IF(W43<AW18,0,1)))
19	=IF(ISERROR(V44/AV19),"F",(IF(V44<AV19,0,1)))	=IF(ISERROR(W44/AW19),"F",(IF(W44<AW19,0,1)))
20	=IF(ISERROR(V45/AV20),"F",(IF(V45<AV20,0,1)))	=IF(ISERROR(W45/AW20),"F",(IF(W45<AW20,0,1)))
21	=IF(ISERROR(V46/AV21),"F",(IF(V46<AV21,0,1)))	=IF(ISERROR(W46/AW21),"F",(IF(W46<AW21,0,1)))
22	=IF(ISERROR(V47/AV22),"F",(IF(V47<AV22,0,1)))	=IF(ISERROR(W47/AW22),"F",(IF(W47<AW22,0,1)))
23	=IF(ISERROR(V48/AV23),"F",(IF(V48<AV23,0,1)))	=IF(ISERROR(W48/AW23),"F",(IF(W48<AW23,0,1)))
24	=IF(ISERROR(V49/AV24),"F",(IF(V49<AV24,0,1)))	=IF(ISERROR(W49/AW24),"F",(IF(W49<AW24,0,1)))
25	=IF(ISERROR(V50/AV25),"F",(IF(V50<AV25,0,1)))	=IF(ISERROR(W50/AW25),"F",(IF(W50<AW25,0,1)))
26	=IF(ISERROR(V51/AV26),"F",(IF(V51<AV26,0,1)))	=IF(ISERROR(W51/AW26),"F",(IF(W51<AW26,0,1)))
27	=IF(ISERROR(V52/AV27),"F",(IF(V52<AV27,0,1)))	=IF(ISERROR(W52/AW27),"F",(IF(W52<AW27,0,1)))
28	=IF(ISERROR(V53/AV28),"F",(IF(V53<AV28,0,1)))	=IF(ISERROR(W53/AW28),"F",(IF(W53<AW28,0,1)))
29	=IF(ISERROR(V54/AV29),"F",(IF(V54<AV29,0,1)))	=IF(ISERROR(W54/AW29),"F",(IF(W54<AW29,0,1)))
30	=IF(ISERROR(V55/AV30),"F",(IF(V55<AV30,0,1)))	=IF(ISERROR(W55/AW30),"F",(IF(W55<AW30,0,1)))

	X	Y	Z	AA
1		Load		1.8
2		App. Stress		=AA1/(AA3*AA4
3		Width		1
4		Thickness		0.1
5		Alpha		5
6		Beta		100
7	=IF(ISERROR(X32/AX7),"F",(IF(X32<AX7,0,1)))	=IF(ISERROR(Y32/AY7),"F",(IF(Y32<AY7,0,1)))	2	
8	=IF(ISERROR(X33/AX8),"F",(IF(X33<AX8,0,1)))	=IF(ISERROR(Y33/AY8),"F",(IF(Y33<AY8,0.1)))	2	
9	=IF(ISERROR(X34/AX9),"F",(IF(X34<AX9,0,1)))	=IF(ISERROR(Y34/AY9),"F",(IF(Y34<AY9,0,1)))	2	
10	=IF(ISERROR(X35/AX10),"F",(IF(X35<AX10,0,1)))	=IF(ISERROR(Y35/AY10),"F",(IF(Y35<AY10,0,1)))	2	
11	=IF(ISERROR(X36/AX11),"F",(IF(X36<AX11,0,1)))	=IF(ISERROR(Y36/AY11),"F",(IF(Y36<AY11,0,1)))	2	
12	=IF(ISERROR(X37/AX12),"F",(IF(X37<AX12,0,1)))	=IF(ISERROR(Y37/AY12),"F",(IF(Y37<AY12,0,1)))	2	
13	=IF(ISERROR(X38/AX13),"F",(IF(X38<AX13,0,1)))	=IF(ISERROR(Y38/AY13),"F",(IF(Y38<AY13,0,1)))	2	
14	=IF(ISERROR(X39/AX14),"F",(IF(X39<AX14,0,1)))	=IF(ISERROR(Y39/AY14),"F",(IF(Y39<AY14,0,1)))	2	
15	=IF(ISERROR(X40/AX15),"F",(IF(X40<AX15,0,1)))	=IF(ISERROR(Y40/AY15),"F",(IF(Y40<AY15,0,1)))	2	
16	=IF(ISERROR(X41/AX16),"F",(IF(X41<AX16,0,1)))	=IF(ISERROR(Y41/AY16),"F",(IF(Y41<AY16,0,1)))	2	
17	=IF(ISERROR(X42/AX17),"F",(IF(X42<AX17,0,1)))	=IF(ISERROR(Y42/AY17),"F",(IF(Y42<AY17,0,1)))	2	
18	=IF(ISERROR(X43/AX18),"F",(IF(X43<AX18,0,1)))	=IF(ISERROR(Y43/AY18),"F",(IF(Y43<AY18,0,1)))	2	
19	=IF(ISERROR(X44/AX19),"F",(IF(X44<AX19,0,1)))	=IF(ISERROR(Y44/AY19),"F",(IF(Y44<AY19,0,1)))	2	
20	=IF(ISERROR(X45/AX20),"F",(IF(X45<AX20,0,1)))	=IF(ISERROR(Y45/AY20),"F",(IF(Y45<AY20,0,1)))	2	
21	=IF(ISERROR(X46/AX21),"F",(IF(X46<AX21,0,1)))	=IF(ISERROR(Y46/AY21),"F",(IF(Y46<AY21,0,1)))	2	
22	=IF(ISERROR(X47/AX22),"F",(IF(X47<AX22,0,1)))	=IF(ISERROR(Y47/AY22),"F",(IF(Y47<AY22,0,1)))	2	
23	=IF(ISERROR(X48/AX23),"F",(IF(X48<AX23,0,1)))	=IF(ISERROR(Y48/AY23),"F",(IF(Y48<AY23,0,1)))	2	
24	=IF(ISERROR(X49/AX24),"F",(IF(X49<AX24,0,1)))	=IF(ISERROR(Y49/AY24),"F",(IF(Y49<AY24,0,1)))	2	
25	=IF(ISERROR(X50/AX25),"F",(IF(X50<AX25,0,1)))	=IF(ISERROR(Y50/AY25),"F",(IF(Y50<AY25,0,1)))	2	
26	=IF(ISERROR(X51/AX26),"F",(IF(X51<AX26,0,1)))	=IF(ISERROR(Y51/AY26),"F",(IF(Y51<AY26,0,1)))	2	
27	=IF(ISERROR(X52/AX27),"F",(IF(X52<AX27,0,1)))	=IF(ISERROR(Y52/AY27),"F",(IF(Y52<AY27,0,1)))	2	
28	=IF(ISERROR(X53/AX28),"F",(IF(X53<AX28,0,1)))	=IF(ISERROR(Y53/AY28),"F",(IF(Y53<AY28,0,1)))	2	
29	=IF(ISERROR(X54/AX29),"F",(IF(X54<AX29,0,1)))	=IF(ISERROR(Y54/AY29),"F",(IF(Y54<AY29,0,1)))	2	
30	=IF(ISERROR(X55/AX30),"F",(IF(X55<AX30,0,1)))	=IF(ISERROR(Y55/AY30),"F",(IF(Y55<AY30,0,1)))	2	

	AB	AC
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6	Intrinsic Strengths	
7	=EXP((LN(-LN(1-AB32))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AC32))+$AA$5*LN($AA$6))/$AA$5)
8	=EXP((LN(-LN(1-AB33))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AC33))+$AA$5*LN($AA$6))/$AA$5)
9	=EXP((LN(-LN(1-AB34))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AC34))+$AA$5*LN($AA$6))/$AA$5)
10	=EXP((LN(-LN(1-AB35))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AC35))+$AA$5*LN($AA$6))/$AA$5)
11	=EXP((LN(-LN(1-AB36))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AC36))+$AA$5*LN($AA$6))/$AA$5)
12	=EXP((LN(-LN(1-AB37))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AC37))+$AA$5*LN($AA$6))/$AA$5)
13	=EXP((LN(-LN(1-AB38))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AC38))+$AA$5*LN($AA$6))/$AA$5)
14	=EXP((LN(-LN(1-AB39))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AC39))+$AA$5*LN($AA$6))/$AA$5)
15	=EXP((LN(-LN(1-AB40))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AC40))+$AA$5*LN($AA$6))/$AA$5)
16	=EXP((LN(-LN(1-AB41))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AC41))+$AA$5*LN($AA$6))/$AA$5)
17	=EXP((LN(-LN(1-AB42))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AC42))+$AA$5*LN($AA$6))/$AA$5)
18	=EXP((LN(-LN(1-AB43))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AC43))+$AA$5*LN($AA$6))/$AA$5)
19	=EXP((LN(-LN(1-AB44))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AC44))+$AA$5*LN($AA$6))/$AA$5)
20	=EXP((LN(-LN(1-AB45))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AC45))+$AA$5*LN($AA$6))/$AA$5)
21	=EXP((LN(-LN(1-AB46))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AC46))+$AA$5*LN($AA$6))/$AA$5)
22	=EXP((LN(-LN(1-AB47))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AC47))+$AA$5*LN($AA$6))/$AA$5)
23	=EXP((LN(-LN(1-AB48))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AC48))+$AA$5*LN($AA$6))/$AA$5)
24	=EXP((LN(-LN(1-AB49))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AC49))+$AA$5*LN($AA$6))/$AA$5)
25	=EXP((LN(-LN(1-AB50))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AC50))+$AA$5*LN($AA$6))/$AA$5)
26	=EXP((LN(-LN(1-AB51))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AC51))+$AA$5*LN($AA$6))/$AA$5)
27	=EXP((LN(-LN(1-AB52))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AC52))+$AA$5*LN($AA$6))/$AA$5)
28	=EXP((LN(-LN(1-AB53))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AC53))+$AA$5*LN($AA$6))/$AA$5)
29	=EXP((LN(-LN(1-AB54))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AC54))+$AA$5*LN($AA$6))/$AA$5)
30	=EXP((LN(-LN(1-AB55))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AC55))+$AA$5*LN($AA$6))/$AA$5)

	AD	AE
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7	=EXP((LN(-LN(1-AD32))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AE32))+$AA$5*LN($AA$6))/$AA$5)
8	=EXP((LN(-LN(1-AD33))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AE33))+$AA$5*LN($AA$6))/$AA$5)
9	=EXP((LN(-LN(1-AD34))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AE34))+$AA$5*LN($AA$6))/$AA$5)
10	=EXP((LN(-LN(1-AD35))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AE35))+$AA$5*LN($AA$6))/$AA$5)
11	=EXP((LN(-LN(1-AD36))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AE36))+$AA$5*LN($AA$6))/$AA$5)
12	=EXP((LN(-LN(1-AD37))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AE37))+$AA$5*LN($AA$6))/$AA$5)
13	=EXP((LN(-LN(1-AD38))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AE38))+$AA$5*LN($AA$6))/$AA$5)
14	=EXP((LN(-LN(1-AD39))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AE39))+$AA$5*LN($AA$6))/$AA$5)
15	=EXP((LN(-LN(1-AD40))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AE40))+$AA$5*LN($AA$6))/$AA$5)
16	=EXP((LN(-LN(1-AD41))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AE41))+$AA$5*LN($AA$6))/$AA$5)
17	=EXP((LN(-LN(1-AD42))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AE42))+$AA$5*LN($AA$6))/$AA$5)
18	=EXP((LN(-LN(1-AD43))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AE43))+$AA$5*LN($AA$6))/$AA$5)
19	=EXP((LN(-LN(1-AD44))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AE44))+$AA$5*LN($AA$6))/$AA$5)
20	=EXP((LN(-LN(1-AD45))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AE45))+$AA$5*LN($AA$6))/$AA$5)
21	=EXP((LN(-LN(1-AD46))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AE46))+$AA$5*LN($AA$6))/$AA$5)
22	=EXP((LN(-LN(1-AD47))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AE47))+$AA$5*LN($AA$6))/$AA$5)
23	=EXP((LN(-LN(1-AD48))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AE48))+$AA$5*LN($AA$6))/$AA$5)
24	=EXP((LN(-LN(1-AD49))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AE49))+$AA$5*LN($AA$6))/$AA$5)
25	=EXP((LN(-LN(1-AD50))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AE50))+$AA$5*LN($AA$6))/$AA$5)
26	=EXP((LN(-LN(1-AD51))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AE51))+$AA$5*LN($AA$6))/$AA$5)
27	=EXP((LN(-LN(1-AD52))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AE52))+$AA$5*LN($AA$6))/$AA$5)
28	=EXP((LN(-LN(1-AD53))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AE53))+$AA$5*LN($AA$6))/$AA$5)
29	=EXP((LN(-LN(1-AD54))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AE54))+$AA$5*LN($AA$6))/$AA$5)
30	=EXP((LN(-LN(1-AD55))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AE55))+$AA$5*LN($AA$6))/$AA$5)

	AF	AG
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7	=EXP((LN(-LN(1-AF32))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AG32))+$AA$5*LN($AA$6))/$AA$5)
8	=EXP((LN(-LN(1-AF33))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AG33))+$AA$5*LN($AA$6))/$AA$5)
9	=EXP((LN(-LN(1-AF34))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AG34))+$AA$5*LN($AA$6))/$AA$5)
10	=EXP((LN(-LN(1-AF35))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AG35))+$AA$5*LN($AA$6))/$AA$5)
11	=EXP((LN(-LN(1-AF36))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AG36))+$AA$5*LN($AA$6))/$AA$5)
12	=EXP((LN(-LN(1-AF37))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AG37))+$AA$5*LN($AA$6))/$AA$5)
13	=EXP((LN(-LN(1-AF38))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AG38))+$AA$5*LN($AA$6))/$AA$5)
14	=EXP((LN(-LN(1-AF39))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AG39))+$AA$5*LN($AA$6))/$AA$5)
15	=EXP((LN(-LN(1-AF40))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AG40))+$AA$5*LN($AA$6))/$AA$5)
16	=EXP((LN(-LN(1-AF41))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AG41))+$AA$5*LN($AA$6))/$AA$5)
17	=EXP((LN(-LN(1-AF42))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AG42))+$AA$5*LN($AA$6))/$AA$5)
18	=EXP((LN(-LN(1-AF43))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AG43))+$AA$5*LN($AA$6))/$AA$5)
19	=EXP((LN(-LN(1-AF44))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AG44))+$AA$5*LN($AA$6))/$AA$5)
20	=EXP((LN(-LN(1-AF45))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AG45))+$AA$5*LN($AA$6))/$AA$5)
21	=EXP((LN(-LN(1-AF46))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AG46))+$AA$5*LN($AA$6))/$AA$5)
22	=EXP((LN(-LN(1-AF47))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AG47))+$AA$5*LN($AA$6))/$AA$5)
23	=EXP((LN(-LN(1-AF48))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AG48))+$AA$5*LN($AA$6))/$AA$5)
24	=EXP((LN(-LN(1-AF49))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AG49))+$AA$5*LN($AA$6))/$AA$5)
25	=EXP((LN(-LN(1-AF50))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AG50))+$AA$5*LN($AA$6))/$AA$5)
26	=EXP((LN(-LN(1-AF51))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AG51))+$AA$5*LN($AA$6))/$AA$5)
27	=EXP((LN(-LN(1-AF52))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AG52))+$AA$5*LN($AA$6))/$AA$5)
28	=EXP((LN(-LN(1-AF53))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AG53))+$AA$5*LN($AA$6))/$AA$5)
29	=EXP((LN(-LN(1-AF54))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AG54))+$AA$5*LN($AA$6))/$AA$5)
30	=EXP((LN(-LN(1-AF55))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AG55))+$AA$5*LN($AA$6))/$AA$5)

	AH	AI
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7	=EXP((LN(-LN(1-AH32))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AI32))+$AA$5*LN($AA$6))/$AA$5)
8	=EXP((LN(-LN(1-AH33))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AI33))+$AA$5*LN($AA$6))/$AA$5)
9	=EXP((LN(-LN(1-AH34))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AI34))+$AA$5*LN($AA$6))/$AA$5)
10	=EXP((LN(-LN(1-AH35))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AI35))+$AA$5*LN($AA$6))/$AA$5)
11	=EXP((LN(-LN(1-AH36))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AI36))+$AA$5*LN($AA$6))/$AA$5)
12	=EXP((LN(-LN(1-AH37))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AI37))+$AA$5*LN($AA$6))/$AA$5)
13	=EXP((LN(-LN(1-AH38))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AI38))+$AA$5*LN($AA$6))/$AA$5)
14	=EXP((LN(-LN(1-AH39))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AI39))+$AA$5*LN($AA$6))/$AA$5)
15	=EXP((LN(-LN(1-AH40))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AI40))+$AA$5*LN($AA$6))/$AA$5)
16	=EXP((LN(-LN(1-AH41))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AI41))+$AA$5*LN($AA$6))/$AA$5)
17	=EXP((LN(-LN(1-AH42))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AI42))+$AA$5*LN($AA$6))/$AA$5)
18	=EXP((LN(-LN(1-AH43))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AI43))+$AA$5*LN($AA$6))/$AA$5)
19	=EXP((LN(-LN(1-AH44))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AI44))+$AA$5*LN($AA$6))/$AA$5)
20	=EXP((LN(-LN(1-AH45))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AI45))+$AA$5*LN($AA$6))/$AA$5)
21	=EXP((LN(-LN(1-AH46))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AI46))+$AA$5*LN($AA$6))/$AA$5)
22	=EXP((LN(-LN(1-AH47))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AI47))+$AA$5*LN($AA$6))/$AA$5)
23	=EXP((LN(-LN(1-AH48))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AI48))+$AA$5*LN($AA$6))/$AA$5)
24	=EXP((LN(-LN(1-AH49))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AI49))+$AA$5*LN($AA$6))/$AA$5)
25	=EXP((LN(-LN(1-AH50))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AI50))+$AA$5*LN($AA$6))/$AA$5)
26	=EXP((LN(-LN(1-AH51))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AI51))+$AA$5*LN($AA$6))/$AA$5)
27	=EXP((LN(-LN(1-AH52))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AI52))+$AA$5*LN($AA$6))/$AA$5)
28	=EXP((LN(-LN(1-AH53))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AI53))+$AA$5*LN($AA$6))/$AA$5)
29	=EXP((LN(-LN(1-AH54))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AI54))+$AA$5*LN($AA$6))/$AA$5)
30	=EXP((LN(-LN(1-AH55))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AI55))+$AA$5*LN($AA$6))/$AA$5)

	AJ	AK
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7	=EXP((LN(-LN(1-AJ32))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AK32))+$AA$5*LN($AA$6))/$AA$5)
8	=EXP((LN(-LN(1-AJ33))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AK33))+$AA$5*LN($AA$6))/$AA$5)
9	=EXP((LN(-LN(1-AJ34))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AK34))+$AA$5*LN($AA$6))/$AA$5)
10	=EXP((LN(-LN(1-AJ35))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AK35))+$AA$5*LN($AA$6))/$AA$5)
11	=EXP((LN(-LN(1-AJ36))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AK36))+$AA$5*LN($AA$6))/$AA$5)
12	=EXP((LN(-LN(1-AJ37))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AK37))+$AA$5*LN($AA$6))/$AA$5)
13	=EXP((LN(-LN(1-AJ38))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AK38))+$AA$5*LN($AA$6))/$AA$5)
14	=EXP((LN(-LN(1-AJ39))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AK39))+$AA$5*LN($AA$6))/$AA$5)
15	=EXP((LN(-LN(1-AJ40))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AK40))+$AA$5*LN($AA$6))/$AA$5)
16	=EXP((LN(-LN(1-AJ41))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AK41))+$AA$5*LN($AA$6))/$AA$5)
17	=EXP((LN(-LN(1-AJ42))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AK42))+$AA$5*LN($AA$6))/$AA$5)
18	=EXP((LN(-LN(1-AJ43))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AK43))+$AA$5*LN($AA$6))/$AA$5)
19	=EXP((LN(-LN(1-AJ44))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AK44))+$AA$5*LN($AA$6))/$AA$5)
20	=EXP((LN(-LN(1-AJ45))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AK45))+$AA$5*LN($AA$6))/$AA$5)
21	=EXP((LN(-LN(1-AJ46))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AK46))+$AA$5*LN($AA$6))/$AA$5)
22	=EXP((LN(-LN(1-AJ47))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AK47))+$AA$5*LN($AA$6))/$AA$5)
23	=EXP((LN(-LN(1-AJ48))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AK48))+$AA$5*LN($AA$6))/$AA$5)
24	=EXP((LN(-LN(1-AJ49))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AK49))+$AA$5*LN($AA$6))/$AA$5)
25	=EXP((LN(-LN(1-AJ50))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AK50))+$AA$5*LN($AA$6))/$AA$5)
26	=EXP((LN(-LN(1-AJ51))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AK51))+$AA$5*LN($AA$6))/$AA$5)
27	=EXP((LN(-LN(1-AJ52))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AK52))+$AA$5*LN($AA$6))/$AA$5)
28	=EXP((LN(-LN(1-AJ53))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AK53))+$AA$5*LN($AA$6))/$AA$5)
29	=EXP((LN(-LN(1-AJ54))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AK54))+$AA$5*LN($AA$6))/$AA$5)
30	=EXP((LN(-LN(1-AJ55))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AK55))+$AA$5*LN($AA$6))/$AA$5)

	AL	AM
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7	=EXP((LN(-LN(1-AL32))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AM32))+$AA$5*LN($AA$6))/$AA$5)
8	=EXP((LN(-LN(1-AL33))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AM33))+$AA$5*LN($AA$6))/$AA$5)
9	=EXP((LN(-LN(1-AL34))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AM34))+$AA$5*LN($AA$6))/$AA$5)
10	=EXP((LN(-LN(1-AL35))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AM35))+$AA$5*LN($AA$6))/$AA$5)
11	=EXP((LN(-LN(1-AL36))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AM36))+$AA$5*LN($AA$6))/$AA$5)
12	=EXP((LN(-LN(1-AL37))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AM37))+$AA$5*LN($AA$6))/$AA$5)
13	=EXP((LN(-LN(1-AL38))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AM38))+$AA$5*LN($AA$6))/$AA$5)
14	=EXP((LN(-LN(1-AL39))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AM39))+$AA$5*LN($AA$6))/$AA$5)
15	=EXP((LN(-LN(1-AL40))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AM40))+$AA$5*LN($AA$6))/$AA$5)
16	=EXP((LN(-LN(1-AL41))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AM41))+$AA$5*LN($AA$6))/$AA$5)
17	=EXP((LN(-LN(1-AL42))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AM42))+$AA$5*LN($AA$6))/$AA$5)
18	=EXP((LN(-LN(1-AL43))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AM43))+$AA$5*LN($AA$6))/$AA$5)
19	=EXP((LN(-LN(1-AL44))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AM44))+$AA$5*LN($AA$6))/$AA$5)
20	=EXP((LN(-LN(1-AL45))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AM45))+$AA$5*LN($AA$6))/$AA$5)
21	=EXP((LN(-LN(1-AL46))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AM46))+$AA$5*LN($AA$6))/$AA$5)
22	=EXP((LN(-LN(1-AL47))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AM47))+$AA$5*LN($AA$6))/$AA$5)
23	=EXP((LN(-LN(1-AL48))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AM48))+$AA$5*LN($AA$6))/$AA$5)
24	=EXP((LN(-LN(1-AL49))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AM49))+$AA$5*LN($AA$6))/$AA$5)
25	=EXP((LN(-LN(1-AL50))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AM50))+$AA$5*LN($AA$6))/$AA$5)
26	=EXP((LN(-LN(1-AL51))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AM51))+$AA$5*LN($AA$6))/$AA$5)
27	=EXP((LN(-LN(1-AL52))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AM52))+$AA$5*LN($AA$6))/$AA$5)
28	=EXP((LN(-LN(1-AL53))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AM53))+$AA$5*LN($AA$6))/$AA$5)
29	=EXP((LN(-LN(1-AL54))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AM54))+$AA$5*LN($AA$6))/$AA$5)
30	=EXP((LN(-LN(1-AL55))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AM55))+$AA$5*LN($AA$6))/$AA$5)

	AN	AO
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7	=EXP((LN(-LN(1-AN32))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AO32))+$AA$5*LN($AA$6))/$AA$5)
8	=EXP((LN(-LN(1-AN33))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AO33))+$AA$5*LN($AA$6))/$AA$5)
9	=EXP((LN(-LN(1-AN34))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AO34))+$AA$5*LN($AA$6))/$AA$5)
10	=EXP((LN(-LN(1-AN35))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AO35))+$AA$5*LN($AA$6))/$AA$5)
11	=EXP((LN(-LN(1-AN36))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AO36))+$AA$5*LN($AA$6))/$AA$5)
12	=EXP((LN(-LN(1-AN37))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AO37))+$AA$5*LN($AA$6))/$AA$5)
13	=EXP((LN(-LN(1-AN38))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AO38))+$AA$5*LN($AA$6))/$AA$5)
14	=EXP((LN(-LN(1-AN39))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AO39))+$AA$5*LN($AA$6))/$AA$5)
15	=EXP((LN(-LN(1-AN40))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AO40))+$AA$5*LN($AA$6))/$AA$5)
16	=EXP((LN(-LN(1-AN41))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AO41))+$AA$5*LN($AA$6))/$AA$5)
17	=EXP((LN(-LN(1-AN42))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AO42))+$AA$5*LN($AA$6))/$AA$5)
18	=EXP((LN(-LN(1-AN43))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AO43))+$AA$5*LN($AA$6))/$AA$5)
19	=EXP((LN(-LN(1-AN44))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AO44))+$AA$5*LN($AA$6))/$AA$5)
20	=EXP((LN(-LN(1-AN45))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AO45))+$AA$5*LN($AA$6))/$AA$5)
21	=EXP((LN(-LN(1-AN46))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AO46))+$AA$5*LN($AA$6))/$AA$5)
22	=EXP((LN(-LN(1-AN47))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AO47))+$AA$5*LN($AA$6))/$AA$5)
23	=EXP((LN(-LN(1-AN48))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AO48))+$AA$5*LN($AA$6))/$AA$5)
24	=EXP((LN(-LN(1-AN49))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AO49))+$AA$5*LN($AA$6))/$AA$5)
25	=EXP((LN(-LN(1-AN50))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AO50))+$AA$5*LN($AA$6))/$AA$5)
26	=EXP((LN(-LN(1-AN51))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AO51))+$AA$5*LN($AA$6))/$AA$5)
27	=EXP((LN(-LN(1-AN52))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AO52))+$AA$5*LN($AA$6))/$AA$5)
28	=EXP((LN(-LN(1-AN53))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AO53))+$AA$5*LN($AA$6))/$AA$5)
29	=EXP((LN(-LN(1-AN54))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AO54))+$AA$5*LN($AA$6))/$AA$5)
30	=EXP((LN(-LN(1-AN55))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AO55))+$AA$5*LN($AA$6))/$AA$5)

	AP	AQ
1		
2		
3		
4		
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6		
7	=EXP((LN(-LN(1-AP32))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AQ32))+$AA$5*LN($AA$6))/$AA$5)
8	=EXP((LN(-LN(1-AP33))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AQ33))+$AA$5*LN($AA$6))/$AA$5)
9	=EXP((LN(-LN(1-AP34))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AQ34))+$AA$5*LN($AA$6))/$AA$5)
10	=EXP((LN(-LN(1-AP35))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AQ35))+$AA$5*LN($AA$6))/$AA$5)
11	=EXP((LN(-LN(1-AP36))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AQ36))+$AA$5*LN($AA$6))/$AA$5)
12	=EXP((LN(-LN(1-AP37))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AQ37))+$AA$5*LN($AA$6))/$AA$5)
13	=EXP((LN(-LN(1-AP38))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AQ38))+$AA$5*LN($AA$6))/$AA$5)
14	=EXP((LN(-LN(1-AP39))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AQ39))+$AA$5*LN($AA$6))/$AA$5)
15	=EXP((LN(-LN(1-AP40))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AQ40))+$AA$5*LN($AA$6))/$AA$5)
16	=EXP((LN(-LN(1-AP41))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AQ41))+$AA$5*LN($AA$6))/$AA$5)
17	=EXP((LN(-LN(1-AP42))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AQ42))+$AA$5*LN($AA$6))/$AA$5)
18	=EXP((LN(-LN(1-AP43))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AQ43))+$AA$5*LN($AA$6))/$AA$5)
19	=EXP((LN(-LN(1-AP44))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AQ44))+$AA$5*LN($AA$6))/$AA$5)
20	=EXP((LN(-LN(1-AP45))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AQ45))+$AA$5*LN($AA$6))/$AA$5)
21	=EXP((LN(-LN(1-AP46))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AQ46))+$AA$5*LN($AA$6))/$AA$5)
22	=EXP((LN(-LN(1-AP47))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AQ47))+$AA$5*LN($AA$6))/$AA$5)
23	=EXP((LN(-LN(1-AP48))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AQ48))+$AA$5*LN($AA$6))/$AA$5)
24	=EXP((LN(-LN(1-AP49))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AQ49))+$AA$5*LN($AA$6))/$AA$5)
25	=EXP((LN(-LN(1-AP50))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AQ50))+$AA$5*LN($AA$6))/$AA$5)
26	=EXP((LN(-LN(1-AP51))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AQ51))+$AA$5*LN($AA$6))/$AA$5)
27	=EXP((LN(-LN(1-AP52))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AQ52))+$AA$5*LN($AA$6))/$AA$5)
28	=EXP((LN(-LN(1-AP53))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AQ53))+$AA$5*LN($AA$6))/$AA$5)
29	=EXP((LN(-LN(1-AP54))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AQ54))+$AA$5*LN($AA$6))/$AA$5)
30	=EXP((LN(-LN(1-AP55))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AQ55))+$AA$5*LN($AA$6))/$AA$5)

	AR	AS
1		
2		
3		
4		
5		
6		
7	=EXP((LN(-LN(1-AR32))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AS32))+$AA$5*LN($AA$6))/$AA$5)
8	=EXP((LN(-LN(1-AR33))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AS33))+$AA$5*LN($AA$6))/$AA$5)
9	=EXP((LN(-LN(1-AR34))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AS34))+$AA$5*LN($AA$6))/$AA$5)
10	=EXP((LN(-LN(1-AR35))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AS35))+$AA$5*LN($AA$6))/$AA$5)
11	=EXP((LN(-LN(1-AR36))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AS36))+$AA$5*LN($AA$6))/$AA$5)
12	=EXP((LN(-LN(1-AR37))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AS37))+$AA$5*LN($AA$6))/$AA$5)
13	=EXP((LN(-LN(1-AR38))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AS38))+$AA$5*LN($AA$6))/$AA$5)
14	=EXP((LN(-LN(1-AR39))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AS39))+$AA$5*LN($AA$6))/$AA$5)
15	=EXP((LN(-LN(1-AR40))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AS40))+$AA$5*LN($AA$6))/$AA$5)
16	=EXP((LN(-LN(1-AR41))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AS41))+$AA$5*LN($AA$6))/$AA$5)
17	=EXP((LN(-LN(1-AR42))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AS42))+$AA$5*LN($AA$6))/$AA$5)
18	=EXP((LN(-LN(1-AR43))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AS43))+$AA$5*LN($AA$6))/$AA$5)
19	=EXP((LN(-LN(1-AR44))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AS44))+$AA$5*LN($AA$6))/$AA$5)
20	=EXP((LN(-LN(1-AR45))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AS45))+$AA$5*LN($AA$6))/$AA$5)
21	=EXP((LN(-LN(1-AR46))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AS46))+$AA$5*LN($AA$6))/$AA$5)
22	=EXP((LN(-LN(1-AR47))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AS47))+$AA$5*LN($AA$6))/$AA$5)
23	=EXP((LN(-LN(1-AR48))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AS48))+$AA$5*LN($AA$6))/$AA$5)
24	=EXP((LN(-LN(1-AR49))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AS49))+$AA$5*LN($AA$6))/$AA$5)
25	=EXP((LN(-LN(1-AR50))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AS50))+$AA$5*LN($AA$6))/$AA$5)
26	=EXP((LN(-LN(1-AR51))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AS51))+$AA$5*LN($AA$6))/$AA$5)
27	=EXP((LN(-LN(1-AR52))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AS52))+$AA$5*LN($AA$6))/$AA$5)
28	=EXP((LN(-LN(1-AR53))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AS53))+$AA$5*LN($AA$6))/$AA$5)
29	=EXP((LN(-LN(1-AR54))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AS54))+$AA$5*LN($AA$6))/$AA$5)
30	=EXP((LN(-LN(1-AR55))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AS55))+$AA$5*LN($AA$6))/$AA$5)

	AT	AU
1		
2		
3		
4		
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6		
7	=EXP((LN(-LN(1-AT32))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AU32))+$AA$5*LN($AA$6))/$AA$5)
8	=EXP((LN(-LN(1-AT33))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AU33))+$AA$5*LN($AA$6))/$AA$5)
9	=EXP((LN(-LN(1-AT34))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AU34))+$AA$5*LN($AA$6))/$AA$5)
10	=EXP((LN(-LN(1-AT35))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AU35))+$AA$5*LN($AA$6))/$AA$5)
11	=EXP((LN(-LN(1-AT36))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AU36))+$AA$5*LN($AA$6))/$AA$5)
12	=EXP((LN(-LN(1-AT37))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AU37))+$AA$5*LN($AA$6))/$AA$5)
13	=EXP((LN(-LN(1-AT38))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AU38))+$AA$5*LN($AA$6))/$AA$5)
14	=EXP((LN(-LN(1-AT39))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AU39))+$AA$5*LN($AA$6))/$AA$5)
15	=EXP((LN(-LN(1-AT40))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AU40))+$AA$5*LN($AA$6))/$AA$5)
16	=EXP((LN(-LN(1-AT41))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AU41))+$AA$5*LN($AA$6))/$AA$5)
17	=EXP((LN(-LN(1-AT42))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AU42))+$AA$5*LN($AA$6))/$AA$5)
18	=EXP((LN(-LN(1-AT43))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AU43))+$AA$5*LN($AA$6))/$AA$5)
19	=EXP((LN(-LN(1-AT44))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AU44))+$AA$5*LN($AA$6))/$AA$5)
20	=EXP((LN(-LN(1-AT45))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AU45))+$AA$5*LN($AA$6))/$AA$5)
21	=EXP((LN(-LN(1-AT46))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AU46))+$AA$5*LN($AA$6))/$AA$5)
22	=EXP((LN(-LN(1-AT47))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AU47))+$AA$5*LN($AA$6))/$AA$5)
23	=EXP((LN(-LN(1-AT48))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AU48))+$AA$5*LN($AA$6))/$AA$5)
24	=EXP((LN(-LN(1-AT49))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AU49))+$AA$5*LN($AA$6))/$AA$5)
25	=EXP((LN(-LN(1-AT50))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AU50))+$AA$5*LN($AA$6))/$AA$5)
26	=EXP((LN(-LN(1-AT51))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AU51))+$AA$5*LN($AA$6))/$AA$5)
27	=EXP((LN(-LN(1-AT52))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AU52))+$AA$5*LN($AA$6))/$AA$5)
28	=EXP((LN(-LN(1-AT53))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AU53))+$AA$5*LN($AA$6))/$AA$5)
29	=EXP((LN(-LN(1-AT54))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AU54))+$AA$5*LN($AA$6))/$AA$5)
30	=EXP((LN(-LN(1-AT55))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AU55))+$AA$5*LN($AA$6))/$AA$5)

	AV	AW
1		
2		
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4		
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6		
7	=EXP((LN(-LN(1-AV32))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AW32))+$AA$5*LN($AA$6))/$AA$5)
8	=EXP((LN(-LN(1-AV33))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AW33))+$AA$5*LN($AA$6))/$AA$5)
9	=EXP((LN(-LN(1-AV34))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AW34))+$AA$5*LN($AA$6))/$AA$5)
10	=EXP((LN(-LN(1-AV35))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AW35))+$AA$5*LN($AA$6))/$AA$5)
11	=EXP((LN(-LN(1-AV36))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AW36))+$AA$5*LN($AA$6))/$AA$5)
12	=EXP((LN(-LN(1-AV37))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AW37))+$AA$5*LN($AA$6))/$AA$5)
13	=EXP((LN(-LN(1-AV38))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AW38))+$AA$5*LN($AA$6))/$AA$5)
14	=EXP((LN(-LN(1-AV39))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AW39))+$AA$5*LN($AA$6))/$AA$5)
15	=EXP((LN(-LN(1-AV40))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AW40))+$AA$5*LN($AA$6))/$AA$5)
16	=EXP((LN(-LN(1-AV41))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AW41))+$AA$5*LN($AA$6))/$AA$5)
17	=EXP((LN(-LN(1-AV42))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AW42))+$AA$5*LN($AA$6))/$AA$5)
18	=EXP((LN(-LN(1-AV43))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AW43))+$AA$5*LN($AA$6))/$AA$5)
19	=EXP((LN(-LN(1-AV44))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AW44))+$AA$5*LN($AA$6))/$AA$5)
20	=EXP((LN(-LN(1-AV45))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AW45))+$AA$5*LN($AA$6))/$AA$5)
21	=EXP((LN(-LN(1-AV46))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AW46))+$AA$5*LN($AA$6))/$AA$5)
22	=EXP((LN(-LN(1-AV47))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AW47))+$AA$5*LN($AA$6))/$AA$5)
23	=EXP((LN(-LN(1-AV48))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AW48))+$AA$5*LN($AA$6))/$AA$5)
24	=EXP((LN(-LN(1-AV49))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AW49))+$AA$5*LN($AA$6))/$AA$5)
25	=EXP((LN(-LN(1-AV50))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AW50))+$AA$5*LN($AA$6))/$AA$5)
26	=EXP((LN(-LN(1-AV51))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AW51))+$AA$5*LN($AA$6))/$AA$5)
27	=EXP((LN(-LN(1-AV52))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AW52))+$AA$5*LN($AA$6))/$AA$5)
28	=EXP((LN(-LN(1-AV53))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AW53))+$AA$5*LN($AA$6))/$AA$5)
29	=EXP((LN(-LN(1-AV54))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AW54))+$AA$5*LN($AA$6))/$AA$5)
30	=EXP((LN(-LN(1-AV55))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AW55))+$AA$5*LN($AA$6))/$AA$5)

	AX	AY
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2		
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6		
7	=EXP((LN(-LN(1-AX32))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AY32))+$AA$5*LN($AA$6))/$AA$5)
8	=EXP((LN(-LN(1-AX33))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AY33))+$AA$5*LN($AA$6))/$AA$5)
9	=EXP((LN(-LN(1-AX34))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AY34))+$AA$5*LN($AA$6))/$AA$5)
10	=EXP((LN(-LN(1-AX35))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AY35))+$AA$5*LN($AA$6))/$AA$5)
11	=EXP((LN(-LN(1-AX36))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AY36))+$AA$5*LN($AA$6))/$AA$5)
12	=EXP((LN(-LN(1-AX37))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AY37))+$AA$5*LN($AA$6))/$AA$5)
13	=EXP((LN(-LN(1-AX38))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AY38))+$AA$5*LN($AA$6))/$AA$5)
14	=EXP((LN(-LN(1-AX39))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AY39))+$AA$5*LN($AA$6))/$AA$5)
15	=EXP((LN(-LN(1-AX40))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AY40))+$AA$5*LN($AA$6))/$AA$5)
16	=EXP((LN(-LN(1-AX41))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AY41))+$AA$5*LN($AA$6))/$AA$5)
17	=EXP((LN(-LN(1-AX42))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AY42))+$AA$5*LN($AA$6))/$AA$5)
18	=EXP((LN(-LN(1-AX43))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AY43))+$AA$5*LN($AA$6))/$AA$5)
19	=EXP((LN(-LN(1-AX44))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AY44))+$AA$5*LN($AA$6))/$AA$5)
20	=EXP((LN(-LN(1-AX45))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AY45))+$AA$5*LN($AA$6))/$AA$5)
21	=EXP((LN(-LN(1-AX46))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AY46))+$AA$5*LN($AA$6))/$AA$5)
22	=EXP((LN(-LN(1-AX47))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AY47))+$AA$5*LN($AA$6))/$AA$5)
23	=EXP((LN(-LN(1-AX48))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AY48))+$AA$5*LN($AA$6))/$AA$5)
24	=EXP((LN(-LN(1-AX49))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AY49))+$AA$5*LN($AA$6))/$AA$5)
25	=EXP((LN(-LN(1-AX50))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AY50))+$AA$5*LN($AA$6))/$AA$5)
26	=EXP((LN(-LN(1-AX51))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AY51))+$AA$5*LN($AA$6))/$AA$5)
27	=EXP((LN(-LN(1-AX52))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AY52))+$AA$5*LN($AA$6))/$AA$5)
28	=EXP((LN(-LN(1-AX53))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AY53))+$AA$5*LN($AA$6))/$AA$5)
29	=EXP((LN(-LN(1-AX54))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AY54))+$AA$5*LN($AA$6))/$AA$5)
30	=EXP((LN(-LN(1-AX55))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AY55))+$AA$5*LN($AA$6))/$AA$5)

	A	B	C	D	E	F
31		**Stress Distribution**				
32		=$AA$2*AZ7	=$AA$2*BA7	=$AA$2*BB7	=$AA$2*BC7	=$AA$2*BD7
33		=$AA$2*AZ8	=$AA$2*BA8	=$AA$2*BB8	=$AA$2*BC8	=$AA$2*BD8
34		=$AA$2*AZ9	=$AA$2*BA9	=$AA$2*BB9	=$AA$2*BC9	=$AA$2*BD9
35		=$AA$2*AZ10	=$AA$2*BA10	=$AA$2*BB10	=$AA$2*BC10	=$AA$2*BD10
36		=$AA$2*AZ11	=$AA$2*BA11	=$AA$2*BB11	=$AA$2*BC11	=$AA$2*BD11
37		=$AA$2*AZ12	=$AA$2*BA12	=$AA$2*BB12	=$AA$2*BC12	=$AA$2*BD12
38		=$AA$2*AZ13	=$AA$2*BA13	=$AA$2*BB13	=$AA$2*BC13	=$AA$2*BD13
39		=$AA$2*AZ14	=$AA$2*BA14	=$AA$2*BB14	=$AA$2*BC14	=$AA$2*BD14
40		=$AA$2*AZ15	=$AA$2*BA15	=$AA$2*BB15	=$AA$2*BC15	=$AA$2*BD15
41		=$AA$2*AZ16	=$AA$2*BA16	=$AA$2*BB16	=$AA$2*BC16	=$AA$2*BD16
42		=$AA$2*AZ17	=$AA$2*BA17	=$AA$2*BB17	=$AA$2*BC17	=$AA$2*BD17
43		=$AA$2*AZ18	=$AA$2*BA18	=$AA$2*BB18	=$AA$2*BC18	=$AA$2*BD18
44		=$AA$2*AZ19	=$AA$2*BA19	=$AA$2*BB19	=$AA$2*BC19	=$AA$2*BD19
45		=$AA$2*AZ20	=$AA$2*BA20	=$AA$2*BB20	=$AA$2*BC20	=$AA$2*BD20
46		=$AA$2*AZ21	=$AA$2*BA21	=$AA$2*BB21	=$AA$2*BC21	=$AA$2*BD21
47		=$AA$2*AZ22	=$AA$2*BA22	=$AA$2*BB22	=$AA$2*BC22	=$AA$2*BD22
48		=$AA$2*AZ23	=$AA$2*BA23	=$AA$2*BB23	=$AA$2*BC23	=$AA$2*BD23
49		=$AA$2*AZ24	=$AA$2*BA24	=$AA$2*BB24	=$AA$2*BC24	=$AA$2*BD24
50		=$AA$2*AZ25	=$AA$2*BA25	=$AA$2*BB25	=$AA$2*BC25	=$AA$2*BD25
51		=$AA$2*AZ26	=$AA$2*BA26	=$AA$2*BB26	=$AA$2*BC26	=$AA$2*BD26
52		=$AA$2*AZ27	=$AA$2*BA27	=$AA$2*BB27	=$AA$2*BC27	=$AA$2*BD27
53		=$AA$2*AZ28	=$AA$2*BA28	=$AA$2*BB28	=$AA$2*BC28	=$AA$2*BD28
54		=$AA$2*AZ29	=$AA$2*BA29	=$AA$2*BB29	=$AA$2*BC29	=$AA$2*BD29
55		=$AA$2*AZ30	=$AA$2*BA30	=$AA$2*BB30	=$AA$2*BC30	=$AA$2*BD30

	AX	AY
1		
2		
3		
4		
5		
6		
7	=EXP((LN(-LN(1-AX32))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AY32))+$AA$5*LN($AA$6))/$AA$5)
8	=EXP((LN(-LN(1-AX33))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AY33))+$AA$5*LN($AA$6))/$AA$5)
9	=EXP((LN(-LN(1-AX34))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AY34))+$AA$5*LN($AA$6))/$AA$5)
10	=EXP((LN(-LN(1-AX35))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AY35))+$AA$5*LN($AA$6))/$AA$5)
11	=EXP((LN(-LN(1-AX36))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AY36))+$AA$5*LN($AA$6))/$AA$5)
12	=EXP((LN(-LN(1-AX37))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AY37))+$AA$5*LN($AA$6))/$AA$5)
13	=EXP((LN(-LN(1-AX38))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AY38))+$AA$5*LN($AA$6))/$AA$5)
14	=EXP((LN(-LN(1-AX39))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AY39))+$AA$5*LN($AA$6))/$AA$5)
15	=EXP((LN(-LN(1-AX40))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AY40))+$AA$5*LN($AA$6))/$AA$5)
16	=EXP((LN(-LN(1-AX41))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AY41))+$AA$5*LN($AA$6))/$AA$5)
17	=EXP((LN(-LN(1-AX42))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AY42))+$AA$5*LN($AA$6))/$AA$5)
18	=EXP((LN(-LN(1-AX43))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AY43))+$AA$5*LN($AA$6))/$AA$5)
19	=EXP((LN(-LN(1-AX44))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AY44))+$AA$5*LN($AA$6))/$AA$5)
20	=EXP((LN(-LN(1-AX45))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AY45))+$AA$5*LN($AA$6))/$AA$5)
21	=EXP((LN(-LN(1-AX46))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AY46))+$AA$5*LN($AA$6))/$AA$5)
22	=EXP((LN(-LN(1-AX47))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AY47))+$AA$5*LN($AA$6))/$AA$5)
23	=EXP((LN(-LN(1-AX48))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AY48))+$AA$5*LN($AA$6))/$AA$5)
24	=EXP((LN(-LN(1-AX49))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AY49))+$AA$5*LN($AA$6))/$AA$5)
25	=EXP((LN(-LN(1-AX50))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AY50))+$AA$5*LN($AA$6))/$AA$5)
26	=EXP((LN(-LN(1-AX51))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AY51))+$AA$5*LN($AA$6))/$AA$5)
27	=EXP((LN(-LN(1-AX52))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AY52))+$AA$5*LN($AA$6))/$AA$5)
28	=EXP((LN(-LN(1-AX53))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AY53))+$AA$5*LN($AA$6))/$AA$5)
29	=EXP((LN(-LN(1-AX54))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AY54))+$AA$5*LN($AA$6))/$AA$5)
30	=EXP((LN(-LN(1-AX55))+$AA$5*LN($AA$6))/$AA$5)	=EXP((LN(-LN(1-AY55))+$AA$5*LN($AA$6))/$AA$5)

	A	B	C	D	E	F
31		**Stress Distribution**				
32		=$AA$2*AZ7	=$AA$2*BA7	=$AA$2*BB7	=$AA$2*BC7	=$AA$2*BD7
33		=$AA$2*AZ8	=$AA$2*BA8	=$AA$2*BB8	=$AA$2*BC8	=$AA$2*BD8
34		=$AA$2*AZ9	=$AA$2*BA9	=$AA$2*BB9	=$AA$2*BC9	=$AA$2*BD9
35		=$AA$2*AZ10	=$AA$2*BA10	=$AA$2*BB10	=$AA$2*BC10	=$AA$2*BD10
36		=$AA$2*AZ11	=$AA$2*BA11	=$AA$2*BB11	=$AA$2*BC11	=$AA$2*BD11
37		=$AA$2*AZ12	=$AA$2*BA12	=$AA$2*BB12	=$AA$2*BC12	=$AA$2*BD12
38		=$AA$2*AZ13	=$AA$2*BA13	=$AA$2*BB13	=$AA$2*BC13	=$AA$2*BD13
39		=$AA$2*AZ14	=$AA$2*BA14	=$AA$2*BB14	=$AA$2*BC14	=$AA$2*BD14
40		=$AA$2*AZ15	=$AA$2*BA15	=$AA$2*BB15	=$AA$2*BC15	=$AA$2*BD15
41		=$AA$2*AZ16	=$AA$2*BA16	=$AA$2*BB16	=$AA$2*BC16	=$AA$2*BD16
42		=$AA$2*AZ17	=$AA$2*BA17	=$AA$2*BB17	=$AA$2*BC17	=$AA$2*BD17
43		=$AA$2*AZ18	=$AA$2*BA18	=$AA$2*BB18	=$AA$2*BC18	=$AA$2*BD18
44		=$AA$2*AZ19	=$AA$2*BA19	=$AA$2*BB19	=$AA$2*BC19	=$AA$2*BD19
45		=$AA$2*AZ20	=$AA$2*BA20	=$AA$2*BB20	=$AA$2*BC20	=$AA$2*BD20
46		=$AA$2*AZ21	=$AA$2*BA21	=$AA$2*BB21	=$AA$2*BC21	=$AA$2*BD21
47		=$AA$2*AZ22	=$AA$2*BA22	=$AA$2*BB22	=$AA$2*BC22	=$AA$2*BD22
48		=$AA$2*AZ23	=$AA$2*BA23	=$AA$2*BB23	=$AA$2*BC23	=$AA$2*BD23
49		=$AA$2*AZ24	=$AA$2*BA24	=$AA$2*BB24	=$AA$2*BC24	=$AA$2*BD24
50		=$AA$2*AZ25	=$AA$2*BA25	=$AA$2*BB25	=$AA$2*BC25	=$AA$2*BD25
51		=$AA$2*AZ26	=$AA$2*BA26	=$AA$2*BB26	=$AA$2*BC26	=$AA$2*BD26
52		=$AA$2*AZ27	=$AA$2*BA27	=$AA$2*BB27	=$AA$2*BC27	=$AA$2*BD27
53		=$AA$2*AZ28	=$AA$2*BA28	=$AA$2*BB28	=$AA$2*BC28	=$AA$2*BD28
54		=$AA$2*AZ29	=$AA$2*BA29	=$AA$2*BB29	=$AA$2*BC29	=$AA$2*BD29
55		=$AA$2*AZ30	=$AA$2*BA30	=$AA$2*BB30	=$AA$2*BC30	=$AA$2*BD30

	G	H	I	J	K
31					
32	=$AA$2*BE7	=$AA$2*BF7	=$AA$2*BG7	=$AA$2*BH7	=$AA$2*BI7
33	=$AA$2*BE8	=$AA$2*BF8	=$AA$2*BG8	=$AA$2*BH8	=$AA$2*BI8
34	=$AA$2*BE9	=$AA$2*BF9	=$AA$2*BG9	=$AA$2*BH9	=$AA$2*BI9
35	=$AA$2*BE10	=$AA$2*BF10	=$AA$2*BG10	=$AA$2*BH10	=$AA$2*BI10
36	=$AA$2*BE11	=$AA$2*BF11	=$AA$2*BG11	=$AA$2*BH11	=$AA$2*BI11
37	=$AA$2*BE12	=$AA$2*BF12	=$AA$2*BG12	=$AA$2*BH12	=$AA$2*BI12
38	=$AA$2*BE13	=$AA$2*BF13	=$AA$2*BG13	=$AA$2*BH13	=$AA$2*BI13
39	=$AA$2*BE14	=$AA$2*BF14	=$AA$2*BG14	=$AA$2*BH14	=$AA$2*BI14
40	=$AA$2*BE15	=$AA$2*BF15	=$AA$2*BG15	=$AA$2*BH15	=$AA$2*BI15
41	=$AA$2*BE16	=$AA$2*BF16	=$AA$2*BG16	=$AA$2*BH16	=$AA$2*BI16
42	=$AA$2*BE17	=$AA$2*BF17	=$AA$2*BG17	=$AA$2*BH17	=$AA$2*BI17
43	=$AA$2*BE18	=$AA$2*BF18	=$AA$2*BG18	=$AA$2*BH18	=$AA$2*BI18
44	=$AA$2*BE19	=$AA$2*BF19	=$AA$2*BG19	=$AA$2*BH19	=$AA$2*BI19
45	=$AA$2*BE20	=$AA$2*BF20	=$AA$2*BG20	=$AA$2*BH20	=$AA$2*BI20
46	=$AA$2*BE21	=$AA$2*BF21	=$AA$2*BG21	=$AA$2*BH21	=$AA$2*BI21
47	=$AA$2*BE22	=$AA$2*BF22	=$AA$2*BG22	=$AA$2*BH22	=$AA$2*BI22
48	=$AA$2*BE23	=$AA$2*BF23	=$AA$2*BG23	=$AA$2*BH23	=$AA$2*BI23
49	=$AA$2*BE24	=$AA$2*BF24	=$AA$2*BG24	=$AA$2*BH24	=$AA$2*BI24
50	=$AA$2*BE25	=$AA$2*BF25	=$AA$2*BG25	=$AA$2*BH25	=$AA$2*BI25
51	=$AA$2*BE26	=$AA$2*BF26	=$AA$2*BG26	=$AA$2*BH26	=$AA$2*BI26
52	=$AA$2*BE27	=$AA$2*BF27	=$AA$2*BG27	=$AA$2*BH27	=$AA$2*BI27
53	=$AA$2*BE28	=$AA$2*BF28	=$AA$2*BG28	=$AA$2*BH28	=$AA$2*BI28
54	=$AA$2*BE29	=$AA$2*BF29	=$AA$2*BG29	=$AA$2*BH29	=$AA$2*BI29
55	=$AA$2*BE30	=$AA$2*BF30	=$AA$2*BG30	=$AA$2*BH30	=$AA$2*BI30

	L	M	N	O	P
31					
32	=$AA$2*BJ7	=$AA$2*BK7	=$AA$2*BL7	=$AA$2*BM7	=$AA$2*BN7
33	=$AA$2*BJ8	=$AA$2*BK8	=$AA$2*BL8	=$AA$2*BM8	=$AA$2*BN8
34	=$AA$2*BJ9	=$AA$2*BK9	=$AA$2*BL9	=$AA$2*BM9	=$AA$2*BN9
35	=$AA$2*BJ10	=$AA$2*BK10	=$AA$2*BL10	=$AA$2*BM10	=$AA$2*BN10
36	=$AA$2*BJ11	=$AA$2*BK11	=$AA$2*BL11	=$AA$2*BM11	=$AA$2*BN11
37	=$AA$2*BJ12	=$AA$2*BK12	=$AA$2*BL12	=$AA$2*BM12	=$AA$2*BN12
38	=$AA$2*BJ13	=$AA$2*BK13	=$AA$2*BL13	=$AA$2*BM13	=$AA$2*BN13
39	=$AA$2*BJ14	=$AA$2*BK14	=$AA$2*BL14	=$AA$2*BM14	=$AA$2*BN14
40	=$AA$2*BJ15	=$AA$2*BK15	=$AA$2*BL15	=$AA$2*BM15	=$AA$2*BN15
41	=$AA$2*BJ16	=$AA$2*BK16	=$AA$2*BL16	=$AA$2*BM16	=$AA$2*BN16
42	=$AA$2*BJ17	=$AA$2*BK17	=$AA$2*BL17	=$AA$2*BM17	=$AA$2*BN17
43	=$AA$2*BJ18	=$AA$2*BK18	=$AA$2*BL18	=$AA$2*BM18	=$AA$2*BN18
44	=$AA$2*BJ19	=$AA$2*BK19	=$AA$2*BL19	=$AA$2*BM19	=$AA$2*BN19
45	=$AA$2*BJ20	=$AA$2*BK20	=$AA$2*BL20	=$AA$2*BM20	=$AA$2*BN20
46	=$AA$2*BJ21	=$AA$2*BK21	=$AA$2*BL21	=$AA$2*BM21	=$AA$2*BN21
47	=$AA$2*BJ22	=$AA$2*BK22	=$AA$2*BL22	=$AA$2*BM22	=$AA$2*BN22
48	=$AA$2*BJ23	=$AA$2*BK23	=$AA$2*BL23	=$AA$2*BM23	=$AA$2*BN23
49	=$AA$2*BJ24	=$AA$2*BK24	=$AA$2*BL24	=$AA$2*BM24	=$AA$2*BN24
50	=$AA$2*BJ25	=$AA$2*BK25	=$AA$2*BL25	=$AA$2*BM25	=$AA$2*BN25
51	=$AA$2*BJ26	=$AA$2*BK26	=$AA$2*BL26	=$AA$2*BM26	=$AA$2*BN26
52	=$AA$2*BJ27	=$AA$2*BK27	=$AA$2*BL27	=$AA$2*BM27	=$AA$2*BN27
53	=$AA$2*BJ28	=$AA$2*BK28	=$AA$2*BL28	=$AA$2*BM28	=$AA$2*BN28
54	=$AA$2*BJ29	=$AA$2*BK29	=$AA$2*BL29	=$AA$2*BM29	=$AA$2*BN29
55	=$AA$2*BJ30	=$AA$2*BK30	=$AA$2*BL30	=$AA$2*BM30	=$AA$2*BN30

	Q	R	S	T	U
31					
32	=$AA$2*BO7	=$AA$2*BP7	=$AA$2*BQ7	=$AA$2*BR7	=$AA$2*BS7
33	=$AA$2*BO8	=$AA$2*BP8	=$AA$2*BQ8	=$AA$2*BR8	=$AA$2*BS8
34	=$AA$2*BO9	=$AA$2*BP9	=$AA$2*BQ9	=$AA$2*BR9	=$AA$2*BS9
35	=$AA$2*BO10	=$AA$2*BP10	=$AA$2*BQ10	=$AA$2*BR10	=$AA$2*BS10
36	=$AA$2*BO11	=$AA$2*BP11	=$AA$2*BQ11	=$AA$2*BR11	=$AA$2*BS11
37	=$AA$2*BO12	=$AA$2*BP12	=$AA$2*BQ12	=$AA$2*BR12	=$AA$2*BS12
38	=$AA$2*BO13	=$AA$2*BP13	=$AA$2*BQ13	=$AA$2*BR13	=$AA$2*BS13
39	=$AA$2*BO14	=$AA$2*BP14	=$AA$2*BQ14	=$AA$2*BR14	=$AA$2*BS14
40	=$AA$2*BO15	=$AA$2*BP15	=$AA$2*BQ15	=$AA$2*BR15	=$AA$2*BS15
41	=$AA$2*BO16	=$AA$2*BP16	=$AA$2*BQ16	=$AA$2*BR16	=$AA$2*BS16
42	=$AA$2*BO17	=$AA$2*BP17	=$AA$2*BQ17	=$AA$2*BR17	=$AA$2*BS17
43	=$AA$2*BO18	=$AA$2*BP18	=$AA$2*BQ18	=$AA$2*BR18	=$AA$2*BS18
44	=$AA$2*BO19	=$AA$2*BP19	=$AA$2*BQ19	=$AA$2*BR19	=$AA$2*BS19
45	=$AA$2*BO20	=$AA$2*BP20	=$AA$2*BQ20	=$AA$2*BR20	=$AA$2*BS20
46	=$AA$2*BO21	=$AA$2*BP21	=$AA$2*BQ21	=$AA$2*BR21	=$AA$2*BS21
47	=$AA$2*BO22	=$AA$2*BP22	=$AA$2*BQ22	=$AA$2*BR22	=$AA$2*BS22
48	=$AA$2*BO23	=$AA$2*BP23	=$AA$2*BQ23	=$AA$2*BR23	=$AA$2*BS23
49	=$AA$2*BO24	=$AA$2*BP24	=$AA$2*BQ24	=$AA$2*BR24	=$AA$2*BS24
50	=$AA$2*BO25	=$AA$2*BP25	=$AA$2*BQ25	=$AA$2*BR25	=$AA$2*BS25
51	=$AA$2*BO26	=$AA$2*BP26	=$AA$2*BQ26	=$AA$2*BR26	=$AA$2*BS26
52	=$AA$2*BO27	=$AA$2*BP27	=$AA$2*BQ27	=$AA$2*BR27	=$AA$2*BS27
53	=$AA$2*BO28	=$AA$2*BP28	=$AA$2*BQ28	=$AA$2*BR28	=$AA$2*BS28
54	=$AA$2*BO29	=$AA$2*BP29	=$AA$2*BQ29	=$AA$2*BR29	=$AA$2*BS29
55	=$AA$2*BO30	=$AA$2*BP30	=$AA$2*BQ30	=$AA$2*BR30	=$AA$2*BS30

	V	W	X	Y	Z
31					
32	=$AA$2*BT7	=$AA$2*BU7	=$AA$2*BV7	=$AA$2*BW7	
33	=$AA$2*BT8	=$AA$2*BU8	=$AA$2*BV8	=$AA$2*BW8	
34	=$AA$2*BT9	=$AA$2*BU9	=$AA$2*BV9	=$AA$2*BW9	
35	=$AA$2*BT10	=$AA$2*BU10	=$AA$2*BV10	=$AA$2*BW10	
36	=$AA$2*BT11	=$AA$2*BU11	=$AA$2*BV11	=$AA$2*BW11	
37	=$AA$2*BT12	=$AA$2*BU12	=$AA$2*BV12	=$AA$2*BW12	
38	=$AA$2*BT13	=$AA$2*BU13	=$AA$2*BV13	=$AA$2*BW13	
39	=$AA$2*BT14	=$AA$2*BU14	=$AA$2*BV14	=$AA$2*BW14	
40	=$AA$2*BT15	=$AA$2*BU15	=$AA$2*BV15	=$AA$2*BW15	
41	=$AA$2*BT16	=$AA$2*BU16	=$AA$2*BV16	=$AA$2*BW16	
42	=$AA$2*BT17	=$AA$2*BU17	=$AA$2*BV17	=$AA$2*BW17	
43	=$AA$2*BT18	=$AA$2*BU18	=$AA$2*BV18	=$AA$2*BW18	
44	=$AA$2*BT19	=$AA$2*BU19	=$AA$2*BV19	=$AA$2*BW19	
45	=$AA$2*BT20	=$AA$2*BU20	=$AA$2*BV20	=$AA$2*BW20	
46	=$AA$2*BT21	=$AA$2*BU21	=$AA$2*BV21	=$AA$2*BW21	
47	=$AA$2*BT22	=$AA$2*BU22	=$AA$2*BV22	=$AA$2*BW22	
48	=$AA$2*BT23	=$AA$2*BU23	=$AA$2*BV23	=$AA$2*BW23	
49	=$AA$2*BT24	=$AA$2*BU24	=$AA$2*BV24	=$AA$2*BW24	
50	=$AA$2*BT25	=$AA$2*BU25	=$AA$2*BV25	=$AA$2*BW25	
51	=$AA$2*BT26	=$AA$2*BU26	=$AA$2*BV26	=$AA$2*BW26	
52	=$AA$2*BT27	=$AA$2*BU27	=$AA$2*BV27	=$AA$2*BW27	
53	=$AA$2*BT28	=$AA$2*BU28	=$AA$2*BV28	=$AA$2*BW28	
54	=$AA$2*BT29	=$AA$2*BU29	=$AA$2*BV29	=$AA$2*BW29	
55	=$AA$2*BT30	=$AA$2*BU30	=$AA$2*BV30	=$AA$2*BW30	

	A	B	C	D	E	F
56		Stress Multipliers				
57		1.015	1.036	1.024	1.008	0.996
58		1.129	1.149	1.097	1.031	0.982
59		1.37	1.31	1.179	1.04	0.942
60		1.708	1.521	1.262	1.013	0.854
61		2.122	1.793	1.34	0.914	0.687
62		2.519	2.204	1.391	0.666	0.404
63		2.536	3.1	0.963	0.205	0.077
64		2.325	3.054	1.045	0.266	0.112
65		2.122	2.124	1.482	0.844	0.649
66		1.753	1.726	1.461	1.169	1.081
67		1.504	1.55	1.457	1.354	1.381
68		1.414	1.484	1.465	1.446	1.529
69		1.414	1.484	1.465	1.446	1.529
70		1.504	1.55	1.457	1.354	1.381
71		1.753	1.726	1.461	1.169	1.081
72		2.122	2.124	1.482	0.844	0.649
73		2.325	3.054	1.045	0.266	0.112
74		2.536	3.1	0.963	0.205	0.077
75		2.519	2.204	1.391	0.666	0.404
76		2.122	1.793	1.34	0.914	0.687
77		1.708	1.521	1.262	1.013	0.854
78		1.37	1.31	1.179	1.04	0.942
79		1.129	1.149	1.097	1.031	0.982
80		1.015	1.036	1.024	1.008	0.996

	G	H	I	J	K
56					
57	0.991	0.991	0.992	0.992	0.99
58	0.958	0.955	0.958	0.955	0.944
59	0.9	0.901	0.913	0.906	0.878
60	0.805	0.836	0.879	0.87	0.811
61	0.652	0.767	0.902	0.882	0.758
62	0.408	0.69	1.13	0.971	0.721
63	0.095	0.33	2.592	0.708	0.514
64	0.134	0.939	2.317	0.892	0.46
65	0.75	0.943	2.089	2.506	0
66	1.244	1.633	2.247	0	0
67	1.638	2.374	0	0	0
68	1.778	2.129	0	0	0
69	1.778	2.129	0	0	0
70	1.638	2.374	0	0	0
71	1.244	1.633	2.247	0	0
72	0.75	0.943	2.089	2.506	0
73	0.134	0.939	2.317	0.892	0.46
74	0.095	0.33	2.592	0.708	0.514
75	0.408	0.69	1.13	0.971	0.721
76	0.652	0.767	0.902	0.882	0.758
77	0.805	0.836	0.879	0.87	0.811
78	0.9	0.901	0.913	0.906	0.878
79	0.958	0.955	0.958	0.955	0.944
80	0.991	0.991	0.992	0.992	0.99

	L	M	N	O	P
56					
57	0.986	0.984	0.984	0.986	0.99
58	0.929	0.919	0.919	0.929	0.944
59	0.843	0.82	0.82	0.843	0.878
60	0.741	0.697	0.697	0.741	0.811
61	0.626	0.547	0.547	0.626	0.758
62	0.49	0.357	0.357	0.49	0.721
63	0.391	0.108	0.108	0.391	0.514
64	0	0	0	0	0.46
65	0	0	0	0	0
66	0	0	0	0	0
67	0	0	0	0	0
68	0	0	0	0	0
69	0	0	0	0	0
70	0	0	0	0	0
71	0	0	0	0	0
72	0	0	0	0	0
73	0	0	0	0	0.46
74	0.391	0.108	0.108	0.391	0.514
75	0.49	0.357	0.357	0.49	0.721
76	0.626	0.547	0.547	0.626	0.758
77	0.741	0.697	0.697	0.741	0.811
78	0.843	0.82	0.82	0.843	0.878
79	0.929	0.919	0.919	0.929	0.944
80	0.986	0.984	0.984	0.986	0.99

	Q	R	S	T	U
56					
57	0.992	0.992	0.991	0.991	0.996
58	0.955	0.958	0.955	0.958	0.982
59	0.906	0.913	0.901	0.9	0.942
60	0.87	0.879	0.836	0.805	0.854
61	0.882	0.902	0.767	0.652	0.687
62	0.971	1.13	0.69	0.408	0.404
63	0.708	2.592	0.33	0.095	0.077
64	0.892	2.317	0.939	0.134	0.112
65	2.506	2.089	0.943	0.75	0.649
66	0	2.247	1.633	1.244	1.081
67	0	0	2.374	1.638	1.381
68	0	0	2.129	1.778	1.529
69	0	0	2.129	1.778	1.529
70	0	0	2.374	1.638	1.381
71	0	2.247	1.633	1.244	1.081
72	2.506	2.089	0.943	0.75	0.649
73	0.892	2.317	0.939	0.134	0.112
74	0.708	2.592	0.33	0.095	0.077
75	0.971	1.13	0.69	0.408	0.404
76	0.882	0.902	0.767	0.652	0.687
77	0.87	0.879	0.836	0.805	0.854
78	0.906	0.913	0.901	0.9	0.942
79	0.955	0.958	0.955	0.958	0.982
80	0.992	0.992	0.991	0.991	0.996

	V	W	X	Y	Z
56					
57	1.008	1.024	1.036	1.015	
58	1.031	1.097	1.149	1.129	
59	1.04	1.179	1.31	1.37	
60	1.013	1.262	1.521	1.708	
61	0.914	1.34	1.793	2.122	
62	0.666	1.391	2.204	2.519	
63	0.205	0.963	3.1	2.536	
64	0.266	1.045	3.054	2.325	
65	0.844	1.482	2.124	2.122	
66	1.169	1.461	1.726	1.753	
67	1.354	1.457	1.55	1.504	
68	1.446	1.465	1.484	1.414	
69	1.446	1.465	1.484	1.414	
70	1.354	1.457	1.55	1.504	
71	1.169	1.461	1.726	1.753	
72	0.844	1.482	2.124	2.122	
73	0.266	1.045	3.054	2.325	
74	0.205	0.963	3.1	2.536	
75	0.666	1.391	2.204	2.519	
76	0.914	1.34	1.793	2.122	
77	1.013	1.262	1.521	1.708	
78	1.04	1.179	1.31	1.37	
79	1.031	1.097	1.149	1.129	
80	1.008	1.024	1.036	1.015	

APPENDIX B STRENGTH DEPENDENCE FOR LINEAR MECHANISTIC COUPLING

In this appendix, the interdependency of the intrinsic strength vectors will be illustrated for an explicitly defined mechanistic coupling model in which the coupling functions, $C_{12}(\sigma_2)$ and $C_{21}(\sigma_1)$ are deterministic linear functions. The same procedure as that used in Chapter IV to prove the dependency of the intrinsic strength vectors for arbitrary deterministic coupling functions and probabilistic independence will be used herein. First, the coupling functions, $C_{12}(\sigma_2)$ and $C_{21}(\sigma_1)$, will be defined and the transformed stress components, σ_1' and σ_2', will be determined in terms of the coupling functions. Second, the reliability will be expressed in terms of the transformed stress components and the joint failure pdf's in both the physical and transformed stress spaces will be derived from the differentiation of the joint failure CDF. Finally, the joint failure pdf in the physical stress space will be compared with the independence criterion given by Equation (4-34) to identify those conditions for which the intrinsic strength vectors are independent for the given coupling model.

The deterministic specimen failure criteria were defined in vector form by Equations (4-1) and (4-2):

$$\underline{F}_1(\sigma_1,\sigma_2;X_1,\Theta_1)=(F_1)_1(\sigma_1,\sigma_2;X_1,\Theta_1)\underline{e}_1+\sigma_2\underline{e}_2$$

$$\underline{F}_2(\sigma_1,\sigma_2;X_2,\Theta_2)=\sigma_1\underline{e}_1+(F_2)_2(\sigma_1,\sigma_2;X_2,\Theta_2)\underline{e}_2$$

For this example, a linear coupling model will be employed. The failure criterion for each failure mode is partitioned into the uniaxial strength, X_i, plus the coupling effect, $C_{ij}\sigma_j$:

$$(\mathcal{F}_1)_1(\sigma_1,\sigma_2;X_1,\Theta_1)=X_1+C_{12}\sigma_2 \tag{B-1}$$

and

$$(\mathcal{F}_2)_2(\sigma_1,\sigma_2;X_2,\Theta_2)=X_2+C_{21}\sigma_1 \tag{B-2}$$

where C_{12} and C_{21} represent constant coupling parameters. These failure criteria are represented for a specimen under an arbitrary biaxial tensile stress is shown in Figure B-1. The uniaxial failure pdf's of the stress components are given by $f_{X_1}(\sigma_1)$ and $f_{X_2}(\sigma_2)$.

Equations (4-16a) and (4-16b) gave two expressions for the composite reliability in terms of the conditional probabilities of the reliability conditions for each failure mode:

$$R_c(\sigma_1,\sigma_2)=\Pr\left\{\left[(\mathcal{F}_1)_1>\sigma_1\right]\middle|\left[(\mathcal{F}_2)_2>\sigma_2\right]\right\}\Pr\left\{(\mathcal{F}_2)_2>\sigma_2\right\}$$

or,

$$R_c(\sigma_1,\sigma_2)=\Pr\left\{\left[(\mathcal{F}_2)_2>\sigma_2\right]\middle|\left[(\mathcal{F}_1)_1>\sigma_1\right]\right\}\Pr\left\{(\mathcal{F}_1)_1>\sigma_1\right\}$$

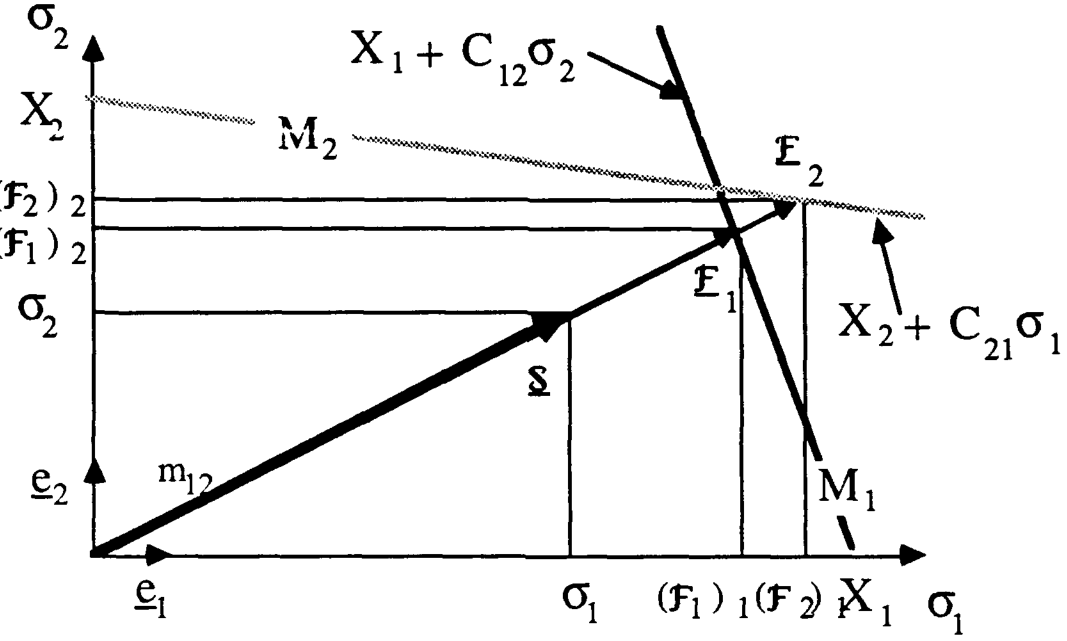

Figure B-1. Representation of the linear coupling model for a specimen under arbitrary biaxial tensile stress.

The scalar components of the strength vectors, $\mathcal{F}_1$ and $\mathcal{F}_2$, $(\mathcal{F}_1)_1$ and $(\mathcal{F}_2)_2$, are related to the uniaxial strengths, X_1 and X_2, by Equations (B-1) and (B-2). Substituting these relations into Equations (4-16a) and (4-16b) and rearranging the inequalities in terms of X_1 and X_2,

$$R_c(\sigma_1,\sigma_2)=\Pr\left\{\left[X_1>\sigma_1-C_{12}\sigma_2\right]\Big|\left[X_2>\sigma_2-C_{21}\sigma_1\right]\right\}\Pr\left\{X_2>\sigma_2-C_{21}\sigma_1\right\} \tag{B-3a}$$

or,

$$R_c(\sigma_1,\sigma_2)=\Pr\left\{\left[X_2>\sigma_2-C_{21}\sigma_1\right]\middle|\left[X_1>\sigma_1-C_{12}\sigma_2\right]\right\}\Pr\left\{X_1>\sigma_1-C_{12}\sigma_2\right\} \tag{B-3b}$$

Rearranging the inequalities in terms of X_1 and X_2 is equivalent to transforming the random variables from the strength vector components to the uniaxial strengths. The composite reliability has been mapped from the physical stress space defined in terms of σ_1 and σ_2 into a transformed stress space defined in terms of transformed stress components σ_1' and σ_2' by

$$\sigma_1'=\sigma_1-C_{12}\sigma_2 \tag{B-4a}$$

and,

$$\sigma_2'=\sigma_2-C_{21}\sigma_1 \tag{B-4b}$$

Differentiating Equations (B-4a) and (B-4b) gives the relations

$$\frac{\partial\sigma_1'}{\partial\sigma_1}=\frac{\partial\sigma_2'}{\partial\sigma_2}=1 \tag{B-5a}$$

$$\frac{\partial\sigma_1'}{\partial\sigma_2}=-C_{12} \tag{B-5b}$$

$$\frac{\partial\sigma_2'}{\partial\sigma_1}=-C_{21} \tag{B-5c}$$

and,

$$\frac{\partial^2 \sigma_1'}{\partial\sigma_1 \partial\sigma_2} = \frac{\partial^2 \sigma_2'}{\partial\sigma_1 \partial\sigma_2} = 0 \tag{B-5d}$$

Equations (B-3a) and (B-3b) may be rewritten in terms of σ_1' and σ_2' as

$$R_c(\sigma_1,\sigma_2) = \Pr\left\{[X_1>\sigma_1'] \,\|\, [X_2>\sigma_2']\right\} \Pr\{X_2>\sigma_2'\} \tag{B-6a}$$

and,

$$R_c(\sigma_1,\sigma_2) = \Pr\left\{[X_2>\sigma_2'] \,\|\, [X_1>\sigma_1']\right\} \Pr\{X_1>\sigma_1'\} \tag{B-6b}$$

The uniaxial strengths , X_1 and X_2, are taken as independent random variables. Therefore, in terms of the transformed stress components, σ_1' and σ_2', the conditional probabilities in Equations (B-6a) and (B-6b) become

$$\Pr\left\{[X_1>\sigma_1'] \,\|\, [X_2>\sigma_2']\right\} = \Pr\{X_1>\sigma_1'\} \tag{B-7}$$

and,

$$\Pr\left\{[X_2>\sigma_2'] \,\|\, [X_1>\sigma_1']\right\} = \Pr\{X_2>\sigma_2'\} \tag{B-8}$$

Substitution of Equations (B-7) and (B-8) into Equations (B-6a) and (B-6b) results in a single logical expression for the composite reliability:

$$R_c(\sigma_1,\sigma_2)=\Pr\{X_1>\sigma_1'\}\Pr\{X_2>\sigma_2'\} \tag{B-9}$$

The two Boolean terms in Equation (B-9) represent uniaxial reliabilities and, hence, the composite reliability in functional form becomes

$$R_c(\sigma_1,\sigma_2)=R_{X_1}(\sigma_1')R_{X_2}(\sigma_2') \tag{B-10}$$

The reliability domain and the joint failure pdf have not actually changed. However, the transformation of the stress space has recast their mathematical expressions in terms of the transformed variables.

In order to determine the joint failure pdf, the composite reliability is converted into the joint CDF for the composite, $F_c(\sigma_1,\sigma_2)$, since $F_c(\sigma_1,\sigma_2)$ is defined as the area integral of the joint failure pdf over the domains of the failure modes in the stress space. Since F = 1–R, Equation (B-10), expressed in terms of CDF's, becomes

$$1-F_c(\sigma_1,\sigma_2)=\{1-F_{X_1}(\sigma_1')\}\{1-F_{X_2}(\sigma_2')\}$$

or,

$$F_c(\sigma_1,\sigma_2)=F_{X_1}(\sigma_1')+F_{X_2}(\sigma_2')-F_{X_1}(\sigma_1')F_{X_2}(\sigma_2') \tag{B-11}$$

Since $R_c(\sigma_1,\sigma_2)$ is invariant for both the physical and transformed stress spaces, $F_c(\sigma_1,\sigma_2)$ is also invariant for the two spaces.

The joint pdf in the transformed stress space, $f_{X_1,X_2}(\sigma_1',\sigma_2')$, may be obtained by differentiating the joint CDF by each of the physical variables, σ_1' and σ_2':

$$f_{X_1,X_2}(\sigma_1',\sigma_2') = \frac{\partial^2}{\partial\sigma_1'\partial\sigma_2'}\left|F_c\right| \tag{B-12}$$

$$= \frac{\partial^2}{\partial\sigma_1'\partial\sigma_2'}\left[F_{X_1}(\sigma_1')F_{X_2}(\sigma_2') - F_{X_1}(\sigma_1') - F_{X_2}(\sigma_2')\right]$$

$$= \frac{\partial}{\partial\sigma_1'}\left[F_{X_1}(\sigma_1')f_{X_2}(\sigma_2') - f_{X_2}(\sigma_2')\right]$$

$$= f_{X_1}(\sigma_1')f_{X_2}(\sigma_2') \tag{B-13}$$

The absolute value is included in Equation (B-12) since the joint pdf must be a positive-valued function; the differentiation of reliability and failure probability result in positive-valued and negative-valued "mirror images" of the joint pdf because $F_c=1-R_c$.

The joint failure pdf in the physical stress space, $f_{(\mathcal{F}_1)_1,(\mathcal{F}_2)_2}(\sigma_1,\sigma_2)$, may similarly be obtained by differentiating the joint CDF by each of the physical variables, σ_1 and σ_2:

$$f_{(\mathcal{F}_1)_1,(\mathcal{F}_2)_2}(\sigma_1,\sigma_2) = \frac{\partial^2}{\partial\sigma_1\partial\sigma_2}\left|F_c\right|$$

$$=\frac{\partial^2}{\partial\sigma_1\partial\sigma_2}\left[F_{X_1}(\sigma_1')F_{X_2}(\sigma_2')-F_{X_1}(\sigma_1')-F_{X_2}(\sigma_2')\right]$$

$$=\left(\frac{\partial\sigma_1'}{\partial\sigma_1}\right)\left(\frac{\partial\sigma_2'}{\partial\sigma_2}\right)f_{X_1}(\sigma_1')f_{X_2}(\sigma_2')+\left(\frac{\partial\sigma_1'}{\partial\sigma_2}\right)\left(\frac{\partial\sigma_2'}{\partial\sigma_1}\right)f_{X_1}(\sigma_1')f_{X_2}(\sigma_2')$$

$$+\left(\frac{\partial\sigma_1'}{\partial\sigma_2}\right)\left(\frac{\partial\sigma_1'}{\partial\sigma_1}\right)\frac{\partial f_{X_1}(\sigma_1')}{\partial\sigma_1}F_{X_2}(\sigma_2')+\left(\frac{\partial\sigma_2'}{\partial\sigma_2}\right)\left(\frac{\partial\sigma_2'}{\partial\sigma_1}\right)F_{X_1}(\sigma_1')\frac{\partial f_{X_2}(\sigma_2')}{\partial\sigma_2}$$

$$+\left(\frac{\partial^2\sigma_1'}{\partial\sigma_1\partial\sigma_2}\right)f_{X_1}(\sigma_1')F_{X_2}(\sigma_2')+\left(\frac{\partial^2\sigma_2'}{\partial\sigma_1\partial\sigma_2}\right)F_{X_1}(\sigma_1')f_{X_2}(\sigma_2')$$

$$-\left(\frac{\partial\sigma_1'}{\partial\sigma_1}\right)\left(\frac{\partial\sigma_1'}{\partial\sigma_2}\right)\frac{\partial f_{X_1}(\sigma_1')}{\partial\sigma_1}-\left(\frac{\partial\sigma_2'}{\partial\sigma_1}\right)\left(\frac{\partial\sigma_2'}{\partial\sigma_2}\right)\frac{\partial f_{X_2}(\sigma_2')}{\partial\sigma_2}$$

$$-\left(\frac{\partial^2\sigma_1'}{\partial\sigma_1\partial\sigma_2}\right)f_{X_1}(\sigma_1')-\left(\frac{\partial^2\sigma_2'}{\partial\sigma_1\partial\sigma_2}\right)f_{X_2}(\sigma_2') \tag{B-14}$$

Substituting the relations of Equations (B-5a) through (B-5d) into Equation (B-13) gives an expression for the joint failure pdf in the physical stress space in terms of the uniaxial failure pdf's and the coupling functions:

$$f_{(\mathcal{F}_1)_1,(\mathcal{F}_2)_2}(\sigma_1,\sigma_2)=(1)(1)f_{X_1}(\sigma_1')f_{X_2}(\sigma_2')+(-C_{12})(-C_{21})f_{X_1}(\sigma_1')f_{X_2}(\sigma_2')$$

$$+(-C_{12})(1)\frac{\partial f_{X_1}(\sigma_1')}{\partial\sigma_1}F_{X_2}(\sigma_2')+(1)(-C_{21})F_{X_1}(\sigma_1')\frac{\partial f_{X_2}(\sigma_2')}{\partial\sigma_2}$$

$$+(0)f_{X_1}(\sigma_1')F_{X_2}(\sigma_2')+(0)F_{X_1}(\sigma_1')f_{X_2}(\sigma_2')$$

$$-(1)(-C_{12})\frac{\partial f_{X_1}(\sigma_1')}{\partial\sigma_1}-(-C_{21})(1)\frac{\partial f_{X_2}(\sigma_2')}{\partial\sigma_2}$$

$$-(0)f_{X_1}(\sigma_1')-(0)f_{X_2}(\sigma_2')$$

$$=f_{X_1}(\sigma_1')f_{X_2}(\sigma_2')+C_{12}C_{21}f_{X_1}(\sigma_1')f_{X_2}(\sigma_2')$$

$$-C_{12}\frac{\partial f_{X_1}(\sigma_1')}{\partial\sigma_1}F_{X_2}(\sigma_2')-C_{21}F_{X_1}(\sigma_1')\frac{\partial f_{X_2}(\sigma_2')}{\partial\sigma_2}$$

$$+C_{12}\frac{\partial f_{X_1}(\sigma_1')}{\partial\sigma_1}+C_{21}\frac{\partial f_{X_2}(\sigma_2')}{\partial\sigma_2} \tag{B-15}$$

In order to determine the interdependency of the physical random variables , $(\mathcal{F}_1)_1$ and $(\mathcal{F}_2)_2$, the conditional probabilities , $f_{(\mathcal{F}_1)_1|(\mathcal{F}_2)_2}(\sigma_1|\sigma_2)$ and $f_{(\mathcal{F}_2)_2|(\mathcal{F}_1)_1}(\sigma_2|\sigma_1)$, must be determined. The conditional probabilities may be found using the relations

$$f_{(\mathcal{F}_1)_1|(\mathcal{F}_2)_2}(\sigma_1|\sigma_2)=\frac{f_{(\mathcal{F}_1)_1,(\mathcal{F}_2)_2}(\sigma_1,\sigma_2)}{f_{X_2}(\sigma_2)} \tag{B-16}$$

and,

$$f_{(\mathcal{F}_2)_2|(\mathcal{F}_1)_1}(\sigma_2|\sigma_1) = \frac{f_{(\mathcal{F}_1)_1,(\mathcal{F}_2)_2}(\sigma_1,\sigma_2)}{f_{X_1}(\sigma_1)} \qquad \text{(B-17)}$$

$(\mathcal{F}_1)_1$ and $(\mathcal{F}_2)_2$, are independent only if their conditional probabilities equal the equivalent uniaxial pdf's or,

$$f_{(\mathcal{F}_1)_1|(\mathcal{F}_2)_2}(\sigma_1|\sigma_2) = f_{X_1}(\sigma_1) \qquad \text{(B-18)}$$

and,

$$f_{(\mathcal{F}_2)_2|(\mathcal{F}_1)_1}(\sigma_2|\sigma_1) = f_{X_2}(\sigma_2) \qquad \text{(B-19)}$$

By substituting Equation (B-16) into Equation (B-18) or Equation (B-17) into Equation (B-19), a single independence criterion for $(\mathcal{F}_1)_1$ and $(\mathcal{F}_2)_2$ may be obtained:

$$f_{(\mathcal{F}_1)_1,(\mathcal{F}_2)_2}(\sigma_1,\sigma_2) = f_{X_1}(\sigma_1) f_{X_2}(\sigma_2) \qquad \text{(B-20)}$$

$(\mathcal{F}_1)_1$ and $(\mathcal{F}_2)_2$ are only independent if there is no mechanistic coupling. The expression for the joint failure pdf in the physical stress space, Equation (B-15) will reduce to the independence criterion for $(\mathcal{F}_1)_1$ and $(\mathcal{F}_2)_2$, Equation (B-20), only if $C_{12} = C_{21} = 0$, which implies that $\sigma_1' = \sigma_1$ and $\sigma_2' = \sigma_2$. If mechanistic coupling is present, i.e., either C_{12} or C_{21} are nonzero, then Equation

(B-15) will not reduce to Equation (B-20), and $(\mathcal{F}_1)_1$ and $(\mathcal{F}_2)_2$ cannot be independent random variables.

APPENDIX C. EXPONENTIAL PARTIAL FRACTION MODEL

In the mathematical modeling of composite materials, it is desired to predict the composite stiffness properties from the elastic properties of the constituent components. Micromechanical analysis, which utilizes the mathematical theory of elasticity as well as energy methods, is frequently used as it treats the multiple phase configurations of a composite with simplified geometrical models. However, for those properties which are geometrically in series (such as transverse and shear moduli), even the simplified models give rise to very complex results. Further simplification of these results for asymptotic stiff fibers leads to relations of the partial fraction form in which the influence of each respective component modulus on the composite modulus is weighted by the volume proportion of the component. This partial fraction weighting is also physically appealing to strength modeling under combined stress in which the strength associated with the dominant failure mode is weakened (i.e., weighted) by the magnitude of the combined stress. Under this consideration, a general form of partial fraction weighting is adopted. The geometric properties of such a form will be examined and the modifications required to reconcile it with the known physical condition of combined stress coupling will be determined.

The general algebraic form of the partial fraction weighting model is

$$y(u)=\frac{a+bu}{c+du} \qquad \text{(C-1)}$$

where a, b, c and d are constant nonnegative coefficients. Equation (C-1) is presented graphically in semi-logarithmic form in Figure C-1.

The partial fraction weighting model as depicted in Figure C-1 is similar in geometric configuration to the graphical representations of combined stress coupling shown in Figures 4-12 and 4-14. There are, however, two significant differences between the representation of the partial fraction weighting model in Figure C-1 and the micromechanical requirements of combined stress coupling as illustrated in Figures 4-12 and 4-14. First, Figure C-1 presents the partial fraction weighting model in semi-logarithmic space while Figures 4-12 and 4-14 represent combined stress coupling in the linear biaxial stress space. Second, Figure C-1 shows that the partial fraction weighting model is a monotonically increasing function of the independent variable, u. Micromechanics requires that the strength associated with one failure mode must be a monotonically decreasing function of the stress component associated with the other failure mode.

These discrepancies may be resolved with an appropriate transformation of the independent variable,u, in Equation (C-1). If Equation (C-1) is recast in terms of the natural logarithm of u, then its geometric configuration in linear space will be identical to that shown in Figure C-1 for semi-logarithmic space and the first discrepancy will be resolved. The second discrepancy will be resolved by recasting Equation (C-1) in terms of the arithmetic inverse of u. In this manner, the graphical representation of Equation (C-1) will be reversed, and the partial fraction weighting model will become a monotonically decreasing function of u. Both discrepancies will be resolved if Equation (C-1) is reformulated in terms of the arithmetic inverse of the natural logarithm of u, or $[\ln u]^{-1}$.

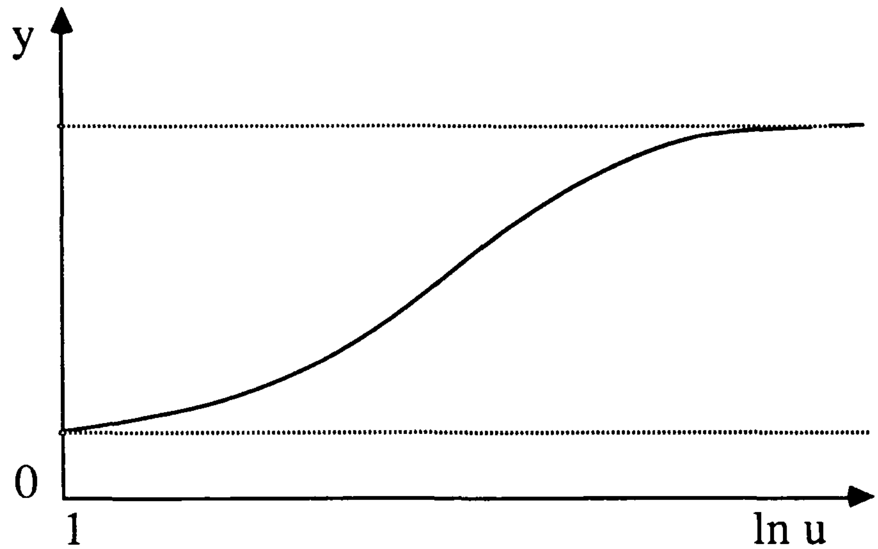

Figure C-1. The general form of the partial fraction weighting model in semi-logarithmic space.

If $v = [\ln u]^{-1}$, then

$$u = \exp\left(\frac{1}{v}\right) \qquad \text{(C-2)}$$

When the variable transformation of Equation (C-2) is substituted into Equation (C-1), both of the discrepancies between the partial fraction weighting model and the micromechanical requirements for combined stress coupling are resolved. After the substitution, Equation (C-1) thus becomes

$$y(v)=\frac{a+(b)\exp\left(\frac{1}{v}\right)}{c+(d)\exp\left(\frac{1}{v}\right)} \tag{C-3}$$

or,

$$y(v)=\frac{(a)\exp\left(-\frac{1}{v}\right)+b}{(c)\exp\left(-\frac{1}{v}\right)+d} \tag{C-4}$$

Equation (C-4) is the exponential form of the partial fraction weighting model and is illustrated in Figure C-2 in linear space. The geometric configuration of the model is now fully reconciled with the representations of combined stress coupling shown in Figures 4-12 and 4-14. Thus, the exponential form of the partial fraction weighting model given by Equation (C-4), may be used for strength modeling under combined stress conditions.

When v equals zero, the exponential terms in Equation (C-4) also equal zero and,

$$y(v=0)=\frac{b}{d} \tag{C-5}$$

As v approaches infinity, the exponential terms in Equation (C-4) approach unity and,

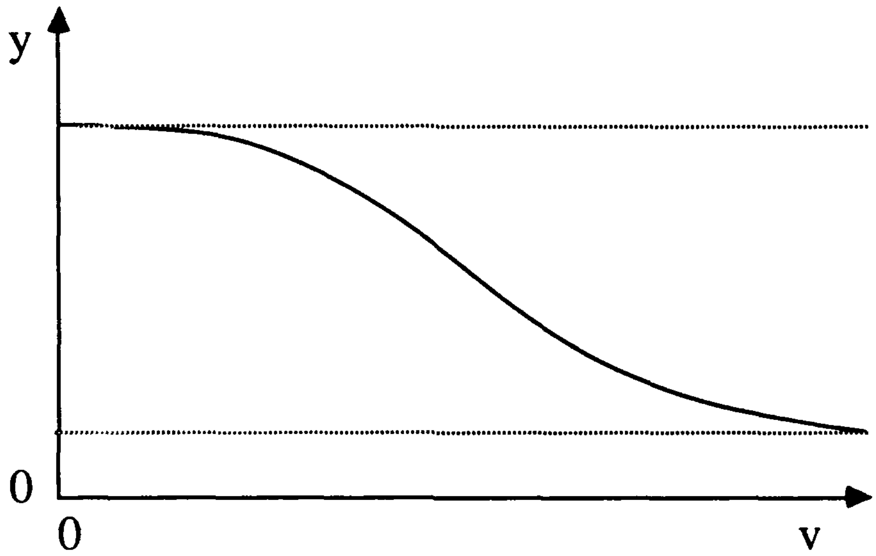

Figure C-2. Exponential form of the partial fraction weighting model in linear space.

$$y(v \to \infty) = \frac{a+b}{c+d} \tag{C-6}$$

The decrease in the longitudinal strength, $(\mathcal{F}_1)_1$, due to transverse loading, σ_2, illustrated in Figure 4-12 may now be modeled using partial fraction weighting of strength under combined stress. Let $y=(\mathcal{F}_1)_1/\sigma_b$ and $v=C_{12}\sigma_2$ where σ_b is the fiber bundle strength and C_{12} is a constant coupling parameter. When $\sigma_2=0$, $v=0$ and $(\mathcal{F}_1)_1$ becomes the uniaxial intrinsic strength, X_1. Therefore, from Equation (C-5),

$$y(v=0) = \frac{b}{d} = \frac{X_1}{\sigma_b} \tag{C-7}$$

Equation (C-7) specifies the values for two of the coefficients in Equation (C-4); the coefficient b is equal to the uniaxial intrinsic strength, X_1, and the coefficient d is equal to the bundle strength, σ_b.

As σ_2 approaches infinity, v approaches infinity and $(\mathcal{F}_1)_1$ approaches the bundle strength, σ_b. If the values for the coefficients b and d obtained from Equation (C-7) are substituted into Equation (C-6), then

$$y(v \to \infty) = \frac{a+b}{c+d} = \frac{a+X_1}{c+\sigma_b} = \frac{\sigma_b}{\sigma_b} \tag{C-8}$$

Equating the numerators in Equation (C-8) and solving for the coefficient a,

$$a = \sigma_b - X_1 \tag{C-9}$$

When the denominators in Equation (C-8) are equated, the coefficient c is equal to zero.

The variable definitions, $y=(\mathcal{F}_1)_1/\sigma_b$ and $v=C_{12}\sigma_2$, and the values for the four coefficients are substituted into Equation (C-4),

$$\frac{(\mathcal{F}_1)_1(\sigma_2)}{\sigma_b} = \frac{(\sigma_b - X_1)\exp\left(-\frac{1}{C_{12}\sigma_2}\right) + X_1}{\sigma_b} \tag{C-10}$$

Solving for the longitudinal strength component, $(\mathcal{F}_1)_1$,

$$(\mathcal{F}_1)_1(\sigma_2) = X_1 - (X_1 - \sigma_b)\exp\left(-\frac{1}{C_{12}\sigma_2}\right) \tag{C-11}$$

which is the model given in Equation (4-38).

The decrease in the transverse strength, $(\mathcal{F}_2)_2$, due to longitudinal loading, σ_1, illustrated in Figure 4-14 may also be modeled using partial fraction weighting of strength under combined stress. Let $y=(\mathcal{F}_2)_2/X_2$ and $v=C_{21}\sigma_1$ where C_{21} is a constant coupling parameter. When $\sigma_1=0$, v=0 and $(\mathcal{F}_2)_2$ becomes the uniaxial intrinsic strength, X_2. Therefore, from Equation (C-5),

$$y(v=0)=\frac{b}{d}=\frac{X_2}{X_2} \tag{C-12}$$

Equation (C-12) specifies the values for two of the coefficients in Equation (C-4); both b and d are equal to the uniaxial intrinsic strength, X_2.

As σ_1 approaches infinity, v approaches infinity and $(\mathcal{F}_2)_2$ approaches zero. If the values for the coefficients b and d obtained from Equation (C-12) are substituted into Equation (C-6), then

$$y(v\rightarrow\infty)=\frac{a+b}{c+d}=\frac{a+X_2}{c+X_2}=0 \tag{C-13}$$

Solving Equation (C-13) for the coefficient a, $a=-X_2$. The value of the coefficient c has no effect on the partial fraction weighting model in the limiting case as σ_1 approaches infinity. Therefore, it will be taken to be the same value as in the previously derived model, or c=0.

The variable definitions, $y=(\mathcal{F}_2)_2/X_2$ and $v=C_{21}\sigma_1$, and the values for the four coefficients are substituted into Equation (C-4),

$$\frac{(\mathcal{F}_2)_2(\sigma_1)}{X_2} = \frac{-X_2 \exp\left(-\frac{1}{C_{21}\sigma_1}\right) + X_2}{X_2} \tag{C-14}$$

Solving for the longitudinal strength component, $(\mathcal{F}_2)_2$,

$$(\mathcal{F}_2)_2(\sigma_1) = X_2 - X_2 \exp\left(-\frac{1}{C_{21}\sigma_1}\right)$$

$$= X_2\left[1 - \exp\left(-\frac{1}{C_{21}\sigma_1}\right)\right] \tag{C-11}$$

which is the model given in Equation (4-39).

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